Name:_____

Mathematics 110 - Fall 2005
Lab Two - Prime Numbers

The **natural numbers** are the positive whole numbers: 1, 2, 3, ...
A **prime number** is a natural number greater than 1 that cannot be written as the product of two natural numbers smaller than itself.

Exercise 1: Finding Primes.

(a) Circle all of the following which are prime. 1 2 3 4 5 6

(b) List all primes less than 100. Hint: there are 25 of them.

Exercise 2: Eliminating Candidates Quickly. In the previous exercise, you may have developed some strategies for quickly deciding that some numbers were not prime.

(a) After the number 2, were there any even primes? Explain why not.

(b) Just by looking at the final digit of a number, we can often determine that the number is not prime. What are the only possible final digits for primes greater than 10? Explain how you know this to be true.

(c) If a number ends in one of the digits you listed in part (b), must that number be prime? If so, explain; if not, give a specific example that shows such numbers need not be prime.

(d) It is possible for a perfect square (such as 1, 4, 9, and so on) to be a prime? If so, give an example; if not, explain why such numbers cannot be prime.

(e) Decide if each of the following is prime and explain your how you know.

(i) 234567892
(ii) 876543215
(iii) 777777777
(iv) 419

Exercise 3: Finding Primes Efficiently.

(a) In checking to see whether 419 was prime in the last exercise, how many possible factors did you have to check? What were these factors?

(b) How can you be sure that you didn't need to check any other factors?

(c) If you wanted to determine if 10001 is prime, how many factors would you have to check? What would these factors be?

Exercise 4: How Common are Primes?

(a) Fill in the table below indicating how many primes there are within each range of numbers shown.

1-10	11-20	21-30	31-40	41-50	51-60	61-70	71-80	81-90	91-100

(b) Find all primes between 521 and 540.

(c) Does it seem that primes become more or less common as you investigate larger numbers?

Exercise 5: Looking Ahead. This exercise leads to questions we'll discuss further at the next class.

(a) (Do this problem by thought alone! Your calculator's limitations are almost certain to give you the wrong answer.) Consider the number we get by multiplying together the first ten primes and then adding 1. This number is $2 \cdot 3 \cdot 5 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 19 \cdot 23 \cdot 29 + 1$. Is this number divisible by 2? by 3? by 5? by any of the first ten primes? Explain how you know.

(b) How many primes do you think there are?

(c) **Twin primes** are two consecutive odd numbers that are both prime, such as 3 and 5 or 17 and 19. How many twin primes do you think there are?