Name:________________________

Mathematics 110 - Fall 2005
Lab One - Fibonacci Numbers

Nearly 800 years ago, an Italian mathematician named Leonardo, who wrote under the name Fibonacci, began studying what we now call the Fibonacci numbers. They have many interesting properties and will arise repeatedly in this course. Even today, modern mathematicians are still learning new things about these special numbers.

Exercise 1: The Basics. This is the problem that originally led Leonardo to study these numbers. Suppose that a particular breed of rabbits has the following properties.

- they cannot reproduce until they are one month old
- they have a gestation period of another month until the babies are born
- each mother always produces two babies, one of each gender

If you start with one pair of baby rabbits at time $t = 0$ months, how many pairs will you have after 1 month? after 2 months? and so on? Fill in the table below to indicate the number of pairs of rabbits alive after each month.

Month	0	1	2	3	4	5	6	7	8	9	10	11	12
Pairs of Rabbits	1												
Fibonacci Number	F_1	F_2	F_3	F_4	F_5	F_6	F_7	F_8	F_9	F_{10}	F_{11}	F_{12}	F_{13}

The numbers in the second row of the table are called the Fibonacci numbers. The first one is denoted F_1, the second F_2, and so forth, with F_k being equal to the number of rabbit pairs after k months.

Exercise 2: Fibonacci Numbers in Nature. These numbers appear in many surprising places in nature. For example, the number of spirals on pine cones, on artichokes, and on many other flowers and plants are Fibonacci numbers. On the attached sheet of paper, count the number of clockwise and counterclockwise spirals on the enlarged photos of a daisy. (The spirals are BETWEEN the computer-generated white lines.)

Exercise 3: The Arithmetic of Fibonacci Numbers.

(a) In completing the table above, you may have noticed a relationship between each Fibonacci number and the previous two numbers.

In words, each Fibonacci number = ____________________ + ____________________

or, in mathematical notation, $F_k =$ ______ + ______.

This last line, along with the fact that $F_1 = 1$ and $F_2 = 1$ is the official way mathematicians define the Fibonacci numbers.

(b) Look at every third Fibonacci number $(F_3, F_6, F_9, \ldots)$. What do they have in common?

(c) Look at every fourth Fibonacci number $(F_4, F_8, F_{12}, \ldots)$. What do you notice about them?

(d) Look at every fifth Fibonacci number $(F_5, F_{10}, \ldots)$. What do you notice about them?

(e) Look at every sixth Fibonacci number $(F_6, F_{12}, \ldots)$. What do you notice about them?

(f) What conjecture would you make about every seventh Fibonacci number? about every tenth? Test your conjectures by finding the fourteenth and twentieth Fibonacci numbers.

Exercise 4: Ratios of Fibonacci Numbers. Use your calculator to fill in the following table with values correct to four decimal places.

F_2/F_1	F_3/F_2	F_4/F_3	F_5/F_4	F_6/F_5	F_7/F_6	F_8/F_7	F_9/F_8	F_{10}/F_9	F_{11}/F_{10}	F_{12}/F_{11}	F_{13}/F_{12}
1.0000	2.0000										

As you go farther along in the table, what seems to be happening to the ratios?

Exercise 5: Looking Ahead at Continued Fractions. Simplify each of the following fractions into a fraction of the form $\frac{p}{q}$, where both p and q are whole numbers.

$$1 + \frac{1}{1} =$$

$$1 + \cfrac{1}{1 + \cfrac{1}{1}} =$$

$$1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1}}} =$$

$$1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1}}}} =$$

$$1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1}}}}} =$$

What do these expressions have to do with the Fibonacci numbers we've been studying?

Now consider a fraction like those above but that continues on forever. Try to compute its exact value. Hint: notice that when this fraction goes on forever, it contains a copy of itself.

Compare this number to the ratios from Exercise 4. We'll look at this again during our next class.