Solutions to Homework Chapters 3.1 and 3.2.

3.1:  5, 7, 20, 22

5) **Bunch of balls.** Most cans of tennis balls contain three balls, so there will NOT be a one-to-one correspondence between cans and balls (unless you have very exclusive, individually packaged tennis balls).

7) **Taking stock.** If the correspondence were not one-to-one, we would have one of the following situations:
   a. A symbol representing no company: This would be a misleading state of affairs. Because no one would buy stock in a company that didn’t exist, you won’t find extraneous symbols in the NY Stock Exchange.
   b. A symbol representing two different companies: This poses a problem if you want to invest in one of the companies but not the other. Furthermore, how would the companies decide to split the money?
   c. A company with no symbol: It’s in the company’s best interest to have an exchange symbol because it makes the buying and selling of stocks easier.

20) **Dorm Life.** There may be a one-to-one correspondence between dorm rooms and students, but we are not guaranteed one. If the rooms were all doubly occupied, we would have more students than rooms. If enough rooms were unoccupied, then we could have more rooms than students. Finally, even if we had the same number of rooms as students, corresponding a student with his or her room wouldn’t necessarily provide a one-to-one correspondence. Half the rooms could be empty, with the other half doubly occupied. Because we have no information about the relative sizes of the two sets, we can’t conclude anything.

22) **Mother and child.** First, let’s assume that we are dealing with all the mothers and children that have lived up to this point, that is, we are dealing with a finite number of mothers and a finite number of children. Because every mother has at least one child, there are at least as many children as mothers. Because some mothers have more than one child, we know that the number of children is strictly larger than the number of mothers. This precludes any one-to-one correspondence between the two sets.

3.2:  2, 3, 4, 5

2) **Au not-so-natural.** The set \{3, 6, 12, 15, 18, \ldots\} is the set of all natural numbers that are multiples of three EXCEPT for 9. The set \{1, 2, 3, 4, 5\} is the set of natural numbers from 1 to 5. The set \{\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots\} is the set of reciprocals of the powers of 2, or the set of powers of \frac{1}{2}. The set \{-1, -2, -3, -4, \ldots\} is the set of all negative integers. The set \{1, 4, 9, 16, 25, 36, 49, 64, 81, 100\} is the set of squares of the numbers from 1 to 10.

3) **Set setup.** The set of natural numbers less than 10 is the set \{1, 2, 3, \ldots, 9\} (OK if you included 10, but strictly speaking 10 is not “less than” 10). The set of all even natural numbers is the set \{2, 4, 6, 8, \ldots\}. The set of solutions to the equation \(x^2 - 4 = 0\) is
the set \{2, -2\}. The set of all reciprocals of the natural numbers is the set \{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots\}.

4) **Little or large.** The first and third sets are finite (the set of natural numbers less than 10 and the set of solutions to the equation \(x^2 - 4 = 0\)). The other sets are infinite (the set of all even natural numbers and the set of all reciprocals of the natural numbers).

5) **A word you can count on.** The **cardinality** of a set is the “number” of elements in the set. If the set is finite, then the **cardinality** of the set really is the number of elements in the set. For infinite sets, we are more likely to speak of two sets having the same cardinality, meaning that their elements can be put into one-to-one correspondence.