1. An object moving around an ellipse has position at time $t \in [0,\ 2\pi]$ given $\mathbf{f}(t) = \overrightarrow{(6\sin(t),\ -4\cos(t))}$.

(1A.) In terms of x and y, what is the equation of that ellipse parameterized by $\mathbf{f}$?

(1B.) In vector form, what is the equation of the line tangent to this ellipse at $t = \pi/4$?

(1C.) Find and solve the equation that gives all $t \in [0,\ 2\pi]$ for which the velocity vector is parallel to the vector $\mathbf{v} = \overrightarrow{(1,1)}$.

(1D.) Find and simplify the function that gives the speed of the object at time t. (Recall, this is just the length of the velocity vector).

(1E.) Use (1D) and your math 105 skills to find all $t \in [0,\ 2\pi]$ at which the speed it maximized. Then identify the positions on the ellipse where this maximum speed is obtained.

2. Suppose that all second partial derivatives of some function $f : \mathbf{R}^2 \to \mathbf{R}$ exist and are continuous.

2A) In terms of f_x, and f_y, what does it mean to say that (a, b) is a *critical point* of f?

2B) In terms of f_{xx}, f_{yy}, and f_{xy}, how does the second derivative test tell us if a critical point (a, b) is a local maximum, local minimum, or saddle point, and under what conditions does the test "fail"?

2C) The graph of $f(x, y) = \sin(x^3) + \sin(y^2)$ accompanies this exam. For this function,

$f_x = 3x^2 \cos(x^3)$ $\quad f_y = 2y \cos(y^2)$, $\quad f_{xx} = -9x^4 \sin(x^3) + 6x \cos(x^3)$ $\quad f_{yy} = -4y^2 \sin(y^2) + 2 \cos(y^2)$, and $f_{xy} = 0$.

Find and label on the graph the three points $\mathbf{a} = \left(\sqrt[3]{\frac{\pi}{2}}, \sqrt{\frac{\pi}{2}}\right)$, $\quad \mathbf{b} = \left(0, \sqrt{\frac{\pi}{2}}\right)$, $\quad \mathbf{c} = \left(-\sqrt[3]{\frac{\pi}{2}}, \sqrt{\frac{\pi}{2}}\right)$.

2D) Verify that each is a critical point of f.

2E) Classify each of them as max/mins/saddle points according to the second derivative test, or explain why the test failed.

2F) Find the value k of the level curve for each of the three points.

2G) Find the directional derivative in the direction of $\mathbf{v} = \overrightarrow{(2, 1)}$ at the point $(-0.5,\ 1)$.

2H) At the point $(-0.5, 1)$, what unit vector points in the direction of maximum increase in f?

3A. Let U be an open connected set in $\mathbf{R}^2$. Let $\mathbf{F} : U \to \mathbf{R}^2$ be a smooth vector field (ie, the four required partial derivatives of $\mathbf{F}$ are continuous). Let R be a simple, connected region in U. What equality does Green's theorem give about certain integrals involving $\mathbf{F}$ and R? Define any additional terms you use in this answer.

3B. Let R be the region in the first quadrant which is above the line $y = 2$, to the right of the line $x = 1$, and below the parabola $y = 11 - x^2$; you may want to sketch the region. Let $\mathbf{F}(x, y) = \overrightarrow{((x-1)y,\ (y-2)x)}$. Illustrate Green's theorem for this function $\mathbf{F}$ on R, that is, show the two integrals the theorem claims are equal, are in fact equal. Show all your work, and make clear what you are doing, especially when it comes to any necessary parameterizations.

4. Let $\mathbf{F}(x,y) = \overrightarrow{(\cos(x)y^2,\ 2\sin(x)y + 3y^2)}$.

4A. Is $\mathbf{F}$ a path independent vector field? Explain.

4B. Find all possible functions $\mathbf{f}$ satisfying $\vec{\nabla}\mathbf{f} = \mathbf{F}$, or explain why there are none.

4C. Find $\iint_R (\ \partial F_2/\partial x\ -\ \partial F_1/\partial y\ )\ dA$, where R is the region in 3B. Make as little work out of it as possible.

5. Consider the circle C of radius 2 on the xy plane centered at the origin. Let R be the region enclosed by this circle. Suppose a vertical wall is constructed all along this circle, and the height z of the wall at any point (x, y) on the circle is $z = 8 - x^2 + y^2$. Let S be the solid object over this circle and inside these walls and having as its "roof" the surface M consisting of the points on the graph of $z = 8 - x^2 + y^2$ which lie over the region R. Let $\mathbf{F}(x, y, z) = \overrightarrow{(x + y,\ y + z,\ xy)}$. A graph of the surface M over the region R is on the last page of this exam.

Find all of the following. For any which involve integrals, write the standard notation for the integral, eg, $\int_C \mathbf{F} \cdot\, d\mathbf{x}$ might be one answer. Then set up and simplify to the extent possible (in particular, remove any traces of vectors!) the actual integrals involved, in terms of s, t, or whatever variables you use. You do not actually have to evaluate any of the integrals.

5A) a parameterization of the circle C, counterclockwise.

5B) The integral representing the amount of work done by $\mathbf{F}$ on an object traveling once around this circle, where $\mathbf{F}$ represents a force field.

5C) A parameterization over some region R_2 of the surface M, with the normal vectors pointing out of the solid (not inward). Clearly state what your R_2 is.

5D) If $\mathbf{F}$ represents the velocity at the point (x, y, z) of a gas moving through this solid, the flux integral of $\mathbf{F}$ over M.

5E) If $\rho(x, y, z) = x + y$ represents the population density of bacteria living on this surface M, the integral representing the total population of bacterial on this surface.

5F) Use your calculator program to find a numerical estimate of the integral in (5E), using $N = 15$; CIRCLE your answer. Also write here what functions you used for Y_1, Y_2, etc, and values for A, B, etc.