Math 206 Section A

Final Exam

100 points

Name: ______________________________

Show all your work to receive full credit for a problem.

There are twelve questions. Questions are printed on both sides of a page.

List of Formulas:

1. $\displaystyle\int_C u\,dL = \int_a^b u(f(t))\ ||f'(t)||\ dt$
2. $\displaystyle\int_C \vec{F}\cdot d\vec{x} = \int_a^b \vec{F}(f(t))\cdot f'(t)\ dt$
3. $\displaystyle\int\int_M g\,d\sigma = \int\int_R g(f(s,t))\ ||f_s(s,t)\times f_t(s,t)||\ dt$
4. $\displaystyle\int\int_M \vec{F}\cdot\vec{n}\,d\sigma = \int\int_R \vec{F}(f(s,t))\cdot(f_s(s,t)\times f_t(s,t))\ ds\,dt$
5. $\displaystyle\int_{\partial R} \vec{F}\cdot d\vec{x} = \int\int_R \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\right)\ dA$
6. $\displaystyle\int\int_{\partial S} \vec{F}\cdot\vec{n}\,d\sigma = \int\int\int_S \mathrm{div}\vec{F}\ dV$
7. $\displaystyle\int_{\partial M} \vec{F}\cdot d\vec{x} = \int\int_M \mathrm{curl}\vec{F}\cdot\vec{n}\ d\sigma$
8. $x = r\cos\theta,\ y = r\sin\theta,\ z = w$
9. $x = \rho\sin\phi\cos\theta,\ y = \rho\sin\phi\sin\theta,\ z = \rho\cos\phi$

1. **(6 points)** Determine the point(s) of discontinuity in the following function. Explain why the point(s) are points of discontinuity. Are the discontinuities removable? Explain.

$$f(x,y) = \frac{e^{x^2+y^2} - 1}{x^2 + y^2}.$$

2. **(6 points)**

(a) Suppose a function $f(x, y)$ is such that

$$\lim_{\substack{(x,y)\to(0,0)\\ x=0}} f(x, y) = -5 \qquad \text{and} \qquad \lim_{\substack{(x,y)\to(0,0)\\ y=0}} f(x, y) = -5\,.$$

What, if anything, can you say about $\displaystyle\lim_{(x,y)\to(0,0)} f(x, y)$? Explain your answer briefly.

(b) Suppose a function $g(x, y)$ is such that $\displaystyle\lim_{(x,y)\to(0,0)} g(x, y) = 5$. What, if anything, can you say about $\displaystyle\lim_{\substack{(x,y)\to(0,0)\\ x=0}} g(x, y)$?

Explain your answer briefly.

3. **(8 points)** Let $f(x, y) = 3xy + y^2$.

(a) At the point $(2, 3)$, what is the direction of the minimum rate of change?

(b) At the point $(2, 3)$, find the rate of change of f in the direction of the vector $(3, -1)$.

4. **(10 points)** Find all the critical points of the function $f(x, y) = x^3 + y^2 - 3x^2 + 10y + 6$. Use the second derivative test to determine whether each critical point is a local minimum, a local maximum or a saddle point.

5. **(10 points)** Let $g(x, y) = x^2 + y$, $f(x, y) = (x, x - y^2)$, and $\vec{a} = (1, 1)$. Let $h = g \circ f$.

(a) Use the chain rule to write a formula for $Dh(\vec{a})$.

(b) Use $h(\vec{a})$ and $Dh(\vec{a})$ to find an approximation for $h(0.99, 1.01)$.

6. **(8 points)** Sketch the region of integration and reverse the order of integration of the following integral. Do not evaluate the integral.

$$\int_0^4 \int_{y/2}^{\sqrt{y}} \ln(x^2 - x^3/3)\, dx\, dy\ .$$

7. **(10 points)** Let $f(x, y, z) = x^2+y^2+z^2+3$ and let S be the solid bounded by $z = 5-x^2-y^2$ and the plane $z = -2$. Write $\int\int\int_S f(x, y, z)\, dV$ as an iterated integral in Cartesian coordinates and cylindrical coordinates. Do not evaluate any of the two integrals.

8. **(8 points)** Evaluate $\int_C \vec{F} \cdot d\vec{x}$ where $\vec{F} = (2xe^{x^2} \sin y, e^{x^2} \cos y)$ and C is the straight line segment from $(0,0)$ to $(1, \pi/2)$.

9. **(8 points)** Evaluate $\int_C \vec{F} \cdot d\vec{x}$ where $\vec{F} = (y, -x)$ and C is the right half of the circle of radius 1 centered at the origin, starting at the point $(0, 1)$ and ending at the point $(0, -1)$.

10. **(9 points)** Evaluate $\int\int_M \vec{F} \cdot \vec{n}\, d\sigma$, where $\vec{F} = (-3x, -3y, -z)$ and M is the part of the surface $z = 2x^2 + 2y^2$ for $0 \leq z \leq 4$, oriented upward.

11. **(8 points)** Let $\vec{F} = (xz, xz^2 + y, x^2)$ and C be the circle $x^2 + y^2 = 1$ in the plane $z = 2$, oriented clockwise when viewed from above.

(a) Find $\text{curl}\,\vec{F}$.

(b) Evaluate $\int_C \vec{F} \cdot d\vec{x}$.

12. **(9 points)** Let $\vec{F} = (xz, 0, y)$ and S be the solid hemisphere bounded by $x^2 + y^2 + z^2 = 4$, $z \geq 0$ and the disk $x^2 + y^2 = 4$, $z = 0$. Use the divergence theorem to evaluate $\int\int_{\partial S} \vec{F} \cdot \vec{n}\, d\sigma$, where ∂S is oriented outward.