Math 206 Section A
Final Exam
100 points

Name: ____________________________________________

Show all your work to receive full credit for a problem.

There are twelve questions. Questions are printed on both sides of a page.

List of Formulas:

1. \[ \int_C u \, dL = \int_a^b u(f(t)) \|f'(t)\| \, dt \]

2. \[ \int_C \vec{F} \cdot d\vec{x} = \int_a^b \vec{F}(f(t)) \cdot f'(t) \, dt \]

3. \[ \int \int_M g \, d\sigma = \int \int_R g(f(s,t)) \|f_s(s,t) \times f_t(s,t)\| \, dt \]

4. \[ \int \int_M \vec{F} \cdot \vec{n} \, d\sigma = \int \int_R \vec{F}(f(s,t)) \cdot (f_s(s,t) \times f_t(s,t)) \, ds \, dt \]

5. \[ \int_{\partial R} \vec{F} \cdot d\vec{x} = \int \int_R \left( \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} \right) \, dA \]

6. \[ \int \int_{\partial S} \vec{F} \cdot \vec{n} \, d\sigma = \int \int \int_S \text{div} \vec{F} \, dV \]

7. \[ \int_{\partial M} \vec{F} \cdot d\vec{x} = \int \int_M \text{curl} \vec{F} \cdot \vec{n} \, d\sigma \]

8. \[ x = r \cos \theta, \quad y = r \sin \theta, \quad z = w \]

9. \[ x = \rho \sin \phi \cos \theta, \quad y = \rho \sin \phi \sin \theta, \quad z = \rho \cos \phi \]
1. **(6 points)** Determine the point(s) of discontinuity in the following function. Explain why the point(s) are points of discontinuity. Are the discontinuities removable? Explain.

\[ f(x, y) = \frac{e^{x^2+y^2} - 1}{x^2 + y^2}. \]
2. (6 points)

(a) Suppose a function \( f(x, y) \) is such that

\[
\lim_{(x,y) \to (0,0)} f(x,y) = -5 \quad \text{and} \quad \lim_{y \to 0} f(x,y) = -5.
\]

What, if anything, can you say about \( \lim_{(x,y) \to (0,0)} f(x,y) \)? Explain your answer briefly.

(b) Suppose a function \( g(x, y) \) is such that \( \lim_{(x,y) \to (0,0)} g(x,y) = 5 \). What, if anything, can you say about \( \lim_{x \to 0} \lim_{y \to 0} g(x,y) \)?

Explain your answer briefly.

3. (8 points) Let \( f(x, y) = 3xy + y^2 \).

(a) At the point (2, 3), what is the direction of the minimum rate of change?

(b) At the point (2, 3), find the rate of change of \( f \) in the direction of the vector (3, -1).
4. (10 points) Find all the critical points of the function \( f(x, y) = x^3 + y^2 - 3x^2 + 10y + 6 \). Use the second derivative test to determine whether each critical point is a local minimum, a local maximum or a saddle point.
5. (10 points) Let \( g(x, y) = x^2 + y \), \( f(x, y) = (x, x - y^2) \), and \( \vec{a} = (1, 1) \). Let \( h = g \circ f \).

(a) Use the chain rule to write a formula for \( Dh(\vec{a}) \).

(b) Use \( h(\vec{a}) \) and \( Dh(\vec{a}) \) to find an approximation for \( h(0.99, 1.01) \).
6. (8 points) Sketch the region of integration and reverse the order of integration of the following integral. Do not evaluate the integral.

\[ \int_{0}^{\sqrt{y}} \int_{y/2}^{4} \ln(x^2 - x^3/3) \, dx \, dy. \]

7. (10 points) Let \( f(x, y, z) = x^2 + y^2 + z^2 + 3 \) and let \( S \) be the solid bounded by \( z = 5 - x^2 - y^2 \) and the plane \( z = -2 \). Write \( \iiint_{S} f(x, y, z) \, dV \) as an iterated integral in Cartesian coordinates and cylindrical coordinates. Do not evaluate any of the two integrals.
8. **(8 points)** Evaluate $\int_{C} \vec{F} \cdot d\vec{x}$ where $\vec{F} = (2xe^{x^2} \sin y, e^{x^2} \cos y)$ and $C$ is the straight line segment from $(0,0)$ to $(1,\pi/2)$. 
9. **(8 points)** Evaluate $\int_C \vec{F} \cdot d\vec{x}$ where $\vec{F} = (y, -x)$ and $C$ is the right half of the circle of radius 1 centered at the origin, starting at the point $(0, 1)$ and ending at the point $(0, -1)$. 
10. (9 points) Evaluate \( \int \int_M \vec{F} \cdot \vec{n} \, d\sigma \), where \( \vec{F} = (-3x, -3y, -z) \) and \( M \) is the part of the surface \( z = 2x^2 + 2y^2 \) for \( 0 \leq z \leq 4 \), oriented upward.
11. **(8 points)** Let $\vec{F} = (xz, xz^2 + y, x^2)$ and $C$ be the circle $x^2 + y^2 = 1$ in the plane $z = 2$, oriented clockwise when viewed from above.

(a) Find curl $\vec{F}$.

(b) Evaluate $\int_C \vec{F} \cdot d\vec{x}$.

12. **(9 points)** Let $\vec{F} = (xz, 0, y)$ and $S$ be the solid hemisphere bounded by $x^2 + y^2 + z^2 = 4$, $z \geq 0$ and the disk $x^2 + y^2 = 4$, $z = 0$. Use the divergence theorem to evaluate $\int_S \vec{F} \cdot \vec{n} d\sigma$, where $\partial S$ is oriented outward.