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December 16 2004

Mathematics 205
Linear Algebra
Final Examination

Mr. Haines

I (5) If A is a 4 x 5 matrix whose null space has dimension 3, what is its rank?

II (5) Give a parametric vector equation of the line through the point $\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$ and parallel to the vector $\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$.

III. (5) Suppose $\mathbf{v_1} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$, $\mathbf{v_2} = \begin{bmatrix} 2 \\ 6 \end{bmatrix}$, and $\mathbf{v_3} = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$ and that W is the subspace of $\Re^2$ consisting of all linear combinations of $\mathbf{v_1}$, $\mathbf{v_2}$, and $\mathbf{v_3}$.

A. What is the dimension of W ?

B. Give a basis for W .

IV. (10) T is a function with rule T($\mathbf{x}$) = A$\mathbf{x}$, where $A = \begin{bmatrix} 0 & 3 & -6 & 6 & 0 & -5 \\ 3 & -7 & 8 & -5 & 3 & 9 \\ 3 & -9 & 12 & -9 & 3 & 15 \end{bmatrix}$

A. What is the domain of T ?

B. What is the codomain of T ?

C. Is T one-to-one? Explain your answer.

V. (5) If $A = \begin{bmatrix} 3 & -1 & -1 \\ 1 & 2 & -4 \\ 1 & 1 & 7 \end{bmatrix}$, calculate A^{-1} without using row reduction or calculating a determinant.

VI. (10) Suppose A = $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$.

A. Give the characteristic polynomial for A .

B. Find the eigenvalues of A .

C. Find a basis for each of the eigenspaces of A .

D. Find a basis for the column space of A.

E. Find a basis for the null space of A.

VII. (10) If $T : \Re^3 \to \Re^3$ reflects points through the plane $x_3 = 0$,

A. Give the standard matrix of the linear transformation T.

B. Give the standard matrix of the linear transformation T^{-1}.

C. What is the determinant of the standard matrix of T ?

VIII. (5) Suppose that A is a 3 x 3 matrix with eigenvalues 10, 4, and -2 and that there is an invertible matrix P and a diagonal matrix D for which $A = PDP^{-1}$. Calculate the determinant of A .

IX. (10) Give an example of

A. A quadratic form in three variables.

B. A linear system of equations in two variables whose solution set is a straight line.

C. A three-dimensional subspace of $\Re^5$. Use correct mathematical notation.

D. A matrix whose null space is a plane in $\Re^3$.

X. (5) $W = \{ (a, b, c, d, e) : c = 0 \}$ is a subspace of $\Re^5$. Calculate the dimension of W.

XI. (5) Suppose an economy has two sectors, Goods and Services. Each year, Goods sells 20% of its output to services and keeps the rest, while Services sells 70% of its output to Goods and keeps the rest. Find equilibrium prices for the annual outputs of the Goods and Services sectors that make each sector's income match its expenditures.

XII. (5) If L is the subspace of $\Re^3$ spanned by $\left\{ \begin{bmatrix} 3 \\ 0 \\ 4 \end{bmatrix} \right\}$, compute the orthogonal projection of the point $\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$ onto L.

XIII. (10) For $x_1^2 + 2x_1x_2 + x_2^2$,

A. Give the matrix of this quadratic form.

B. Give a change of variables that transforms this quadratic form into a quadratic form with no cross-product term.

C. What is this new quadratic form that results from this change of variable?

XIV. (10) TRUE OR FALSE? (Don't guess! The number of incorrect responses will be subtracted from the number of correct ones. Thus, random guessing earns you no points at all.)

________ 1. If AB = AC and A is invertible, then A = C for all matrices A, B, and C.

________ 2. A quadratic form is a special kind of linear transformation.

________ 3. The kernel of a linear transformation is a vector space.

________ 4. If $\mathbf{v_1}$ is a multiple of $\mathbf{v_2}$, then the set $\{ \mathbf{v_1}, \mathbf{v_2} \}$ is linearly independent.

________ 5. If a matrix is diagonalizable, then it has an inverse.

________ 6. $\Re^3$ is a subspace of $\Re^5$.

________ 7. The null space of a 4 x 5 matrix is a subspace of $\Re^4$.

________ 8. Every vector space of dimension 72 must have a basis with 72 elements.

________ 9. It is possible to have two matrices A and B that are invertible, and their product is also invertible.

_______ 10. It is possible to have a vector whose length is -1 .