MATH 205A,B LINEAR ALGEBRA - PROF. P. WONG

FINAL EXAM - DECEMBER 15, 2015

NAME: **Section:**(Circle one) A(8 : 00) B(9 : 30)

Instruction: Read each question carefully. Explain **ALL** your work and give reasons to support your answers.

Advice: DON'T spend too much time on a single problem.

Problems	**Maximum Score**	**Your Score**
1.	21	
2.	23	
3.	22	
4.	19	
5.	22	
6.	21	
7.	22	
Total	150	

1. Let

$$A = \begin{bmatrix} 1 & -5 & -7 \\ -3 & 7 & 5 \end{bmatrix} \qquad \text{and} \qquad \vec{b} = \begin{bmatrix} -2 \\ -2 \end{bmatrix}.$$

(a)(7 pts) Solve the system $A\vec{x} = \vec{b}$.

(b)(7 pts) Find a basis for the column space Col$A$ of A.

(c)(7 pts) Determine the rank of A. Justify your answer.

2. Let

$$A = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 3 & 3 \\ 0 & 3 & 3 \end{bmatrix}.$$

(a)(6 pts) Find the characteristic polynomial of A.

(b)(10 pts) Find the eigenvalues of A and their corresponding eigenspaces.

(c)(7 pts) Is A diagonalizable? Explain.

3. Let

$$A = \begin{bmatrix} 1 & -1 & 2 \\ -1 & 2 & -1 \end{bmatrix}.$$

(a)(7 pts) Find a basis for the null space Nul$A$ of A.

(b)(7 pts) Find a basis for the column space Col$A$ of A.

(c)(8 pts) If $T : \mathbb{R}^3 \to \mathbb{R}^2$ is the linear transformation defined by $T(\vec{x}) = A\vec{x}$, determine whether T is onto. Justify your answer.

4. Suppose that A is a 3×3 matrix with eigenvalues 1 and -2 and that $\dim \mathrm{Col}(A - I) = 1$.

(a)(7 pts) Determine whether A is diagonalizable. Explain. (Hint: what are the dimensions of the corresponding eigenspaces?)

(b)(6 pts) Find $\det(A + 2I)$, the determinant of the matrix $(A + 2I)$. Justify your answer.

(c)(6 pts) Find the characteristic polynomial of A.

5. Let

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}.$$

Two of the three eigenvalues of A are 1 and 2.

(a)(7 pts) Show that $\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$ is an eigenvector of A. Find the corresponding eigenvalue.

(b)(8 pts) Find the eigenspaces corresponding to $\lambda = 1$ and $\lambda = 2$.

(c)(7 pts) Find an orthogonal matrix P and a diagonal matrix D such that $A = PDP^{-1}$.

6. Let $\vec{u} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$, $\vec{y} = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$ and $W = \text{Span}\{\vec{u}, \vec{v}\}$.

(a)(7 pts) Find an orthogonal basis for W.

(b)(7 pts) Find the closest point in W to $\vec{y}$.

(c)(7 pts) What is the shortest distance between $\vec{y}$ and W?

7. Let $T : \mathbb{P}_2 \to \mathbb{P}_2$ be given by

$$T(a + bx + cx^2) = a + b(x + 1) + c(x + 1)^2$$

where a, b, c are real numbers.

(a)(8 pts) Show that T is a linear transformation.

(b)(7 pts) Find Ker$T$, the kernel of T. Is T one-to-one? Justify your answer.

(c)(7 pts) Let $\mathcal{B} = \{1, x, x^2\}$ be a basis for $\mathbb{P}_2$. Find the $\mathcal{B}$ matrix of T, i.e., the matrix M such that $[T(\mathbf{p}(x))]_{\mathcal{B}} = M[\mathbf{p}(x)]_{\mathcal{B}}$.