Name:______________________________

Final Exam

Show all your work to receive full credit for a problem.

1. Let

$$f(x, y) = \frac{x + y^2}{2x + y}.$$

(a) Find the domain of f.

(b) Show that $(0, 0)$ is a non-removable discontinuity.

2. Let $g(x, y, z) = x^2 + y^2 + z^2$.

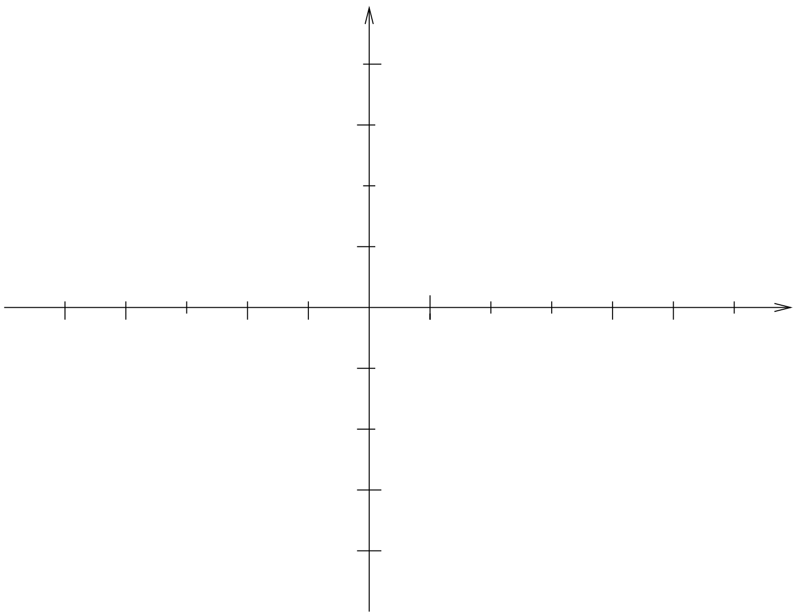
   (a) Describe the level surfaces of g. (You can do this in many ways: verbally, through equations, or through pictures, for example).

   (b) Find a parametrization for the level surface corresponding to $c = 9$.

   (c) Find the equation for the plane tangent to the surface corresponding to $c = 9$ at the point $(1, 5, -1)$.

3. Let $\mathbf{F}$ be the vector field defined by $\mathbf{F} = x\mathbf{i} - y\mathbf{j}$. Let C be the path that is made up of two line segments, the first goes from $(0, 2)$ to $(0, 0)$ and the second from $(0, 0)$ to $(2, 2)$.

(a) Draw six vectors in the vector field $\mathbf{F}$ in the coordinate axes given below.



(b) Evaluate

$$\int_C \mathbf{F} \cdot d\mathbf{x}$$

4. Let $f(x, y)$ be a function that is differentiable everywhere. At a certain point P in the xy-plane, the directional derivative of f in the direction of $\mathbf{i} - \mathbf{j}$ is $\sqrt{2}$ and the directional derivative of f in the direction of $\mathbf{i} + \mathbf{j}$ is $3\sqrt{2}$.

   (a) What is the gradient of f at P?

   (b) What is the maximum directional derivative of f at P?

5. Let f be a differentiable function such that, at the critical point $\mathbf{a}$, the Hessian is

$$Hf(\mathbf{a}) = \begin{pmatrix} -6 & 0 & 1 \\ 0 & -3 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

Determine whether $\mathbf{a}$ is a local maximum, a local minimum, or a saddle point.

6. Let S be the solid bounded above by $z = e^{1-x^2-y^2}$, and below by $z = 1$, and let $\mathbf{G} = x\mathbf{i} + y\mathbf{j} + (2 - 2z)\mathbf{k}$.

(a) What is ∂S? (You don't need to draw it).

(b) Assuming that ∂S is oriented by an outward pointing normal, use the divergence theorem to calculate

$$\iint_{\partial S} \mathbf{G} \cdot \mathbf{n} d\sigma$$

(c) Give a physical interpretation of the result you obtained in part (b).

7. Let $f(x, y) = x^2 - 3y^2$ and $\mathbf{g}(s, t) = (st, s + t^2)$.

(a) Calculate $D(f \circ \mathbf{g})(1, -1)$

(b) Suppose $\mathbf{a}$ is a point very close to $(1, -1)$. Explain how you would use part (a) to find an approximate value for $(f \circ \mathbf{g})(\mathbf{a})$.

8. Let M be the surface defined by $x^2 + y^2 + 5z = 1$, $z \geq 0$, oriented by upward normal. Let $\mathbf{F}$ be the vector field $\mathbf{F} = xz\mathbf{i} + yz\mathbf{j} + (x^2 + y^2)\mathbf{k}$.

(a) Describe ∂M, the boundary of M.

(b) Use Stokes's Theorem to show that $\oint_{\partial M} \mathbf{F} \cdot d\mathbf{x} = 0$.

(c) Does your result for part (b) imply that $\mathbf{F}$ is path independent? Explain why or why not.

9. Use triple integrals to verify that the volume of a ball of radius r is $4\pi r^3/3$.