1. Let $B = \begin{bmatrix} 1 & 3 & 11 \\ 4 & -2 & 2 \\ 3 & 1 & 9 \end{bmatrix}$. (1A) Find a basis for $\text{Col}(B)^\perp$.

(1B) Label the row vectors of B as $\mathbf{r}_1$, $\mathbf{r}_2$, and $\mathbf{r}_3$. It's clear that $\{\mathbf{r}_2,\ \mathbf{r}_3\}$ form a LI set. But $\mathbf{r}_1$ is a LC of $\mathbf{r}_2$ and $\mathbf{r}_3$. Find that LC using appropriate techniques. Then verify that it's right (writing your vectors horizontally) by direct evaluation of your LC.

2. The points $(-2, 29)$, $(1, 8)$, $(4, 23)$ and $(7, 74)$ all belong to a parabola of the form $\beta_2 x^2 + \beta_1 x + \beta_0$. Find β_2, β_1 and β_0 by setting up and solving an appropriate system of linear equations. Show all work and any matrices and their rrefs that you use. (This is not a best-fit problem!)

3. No single parabola of the form $\beta_2 x^2 + \beta_1 x + \beta_0$ can contain all the points $(-2, 29)$, $(1, 8)$, $(4, -13)$ and $(7, 74)$; you do not have to prove this. However: Find the best fit (least squares) parabola of the form $\beta_2 x^2 + \beta_1 x + \beta_0$ which passes through $(-2, 29)$, $(1, 8)$, $(4, -13)$ and $(7, 74)$. Express your answer in decimals.

4. Even though the parabola in problem (2) contains three out of the four points $(-2, 29)$, $(1, 8)$, $(4, -13)$ and $(7, 74)$, it is not the best fit parabola. Show why it isn't by computing the sum of the squares (SOS) of the residuals it produces and the SOS for the best fit parabola in (3) and comparing them.

5. Let $A = \begin{bmatrix} 1 & 1 & 7 & 2 & 2 \\ 3 & 0 & 9 & 3 & 4 \\ -3 & 1 & -5 & -2 & 3 \\ 2 & 2 & 14 & 4 & 2 \end{bmatrix}$ and $Z = \begin{bmatrix} 4 & 5 & 3 & 4 \\ 5 & 6 & 5 & -3 \\ 10 & -3 & 9 & -106 \\ 4 & 10 & 2 & 44 \end{bmatrix}$ respectively; then their rref's are $\begin{bmatrix} 1 & 0 & 3 & 1 & 0 \\ 0 & 1 & 4 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ and $\begin{bmatrix} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 1 & -5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$. Let $\mathbf{a}_1, \ldots, \mathbf{a}_5$ be the column vectors of A and $\mathbf{z}_1, \ldots, \mathbf{z}_4$ be the column vectors of Z.

5A) Find a basis $\mathcal{M}$ for Col(A). (Use the names $\mathbf{a}_1$ etc; don't write out the vectors explicitly).

5B) Find a basis $\mathcal{N}$ for Col(Z). (Express $\mathcal{N}$ in terms of the $\mathbf{z}_i$'s).

5C) In this problem, Col(A) and Col(Z) are equal. What 4-by-6 superaugmented matrix represents the problem of finding how to express each of the basis elements in $\mathcal{N}$ as a LC of those in $\mathcal{M}$?

5D) Complete problem 5C to actually find those LC's. Express each $\mathbf{z}_i$ in "$[\ ]_{\mathcal{M}}$" notation and *circle* your answers.

this problem continues next page

5E) Find the corresponding change of basis matrix $\mathcal{P}$ from $\mathcal{N}$ to $\mathcal{M}$.

5F) Express $\mathbf{z}_4$ in "$[\ ]_{\mathcal{N}}$" notation.

5G) Use $\mathcal{P}$ and the answer to (5F) to write $\mathbf{z}_4$ as a LC of the basis $\mathcal{M}$ elements.

5H) Compute that LC using the appropriate matrix product to verify its correctness. (What is that product?)

6. Let $A = \begin{bmatrix} -1 & -5 & 3 & 9 \\ -48 & -40 & 24 & 92 \\ 94 & 70 & -42 & -166 \\ -48 & -40 & 24 & 92 \end{bmatrix}$; its rref is $\begin{bmatrix} 1 & 0 & 0 & -1/2 \\ 0 & 1 & -3/5 & -17/10 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Here are two facts: 1) $\lambda = 4$ is an eigenvalue of A. 2) $\mathbf{u} = \begin{bmatrix} 1 \\ 0 \\ 2 \\ 0 \end{bmatrix}$ is an eigenvector for A.

6A) Find a basis for Nul(A) (see the rref above!)

6B) Find a basis for the eigenspace for $\lambda = 4$.

6C) What is the eigenvalue for the eigenvector $\mathbf{u}$?

6D) Show that A can be diagonalized by finding the appropriate P, D and P^{-1}.

6E) Find the characteristic polynomial of A.

7. Let $S = \{x^2 + 4x + 1,\ 4x^2 - 5x + 1,\ 2x^2 + 8x + 2\}$ be a subset of $\mathbb{P}_2$.

7A). Explain why or why not S is a LI set.

7B). Find any/all conditions on a, b, and c which guarantee that $ax^2 + bx + c$ is in the span of S.

8. Suppose T is a linear transformation from $\mathbb{R}^2 \to \mathbb{R}^2$ and

$$T\left(\begin{bmatrix}6\\5\end{bmatrix}\right) = \begin{bmatrix}10\\30\end{bmatrix} \text{ and } T\left(\begin{bmatrix}5\\4\end{bmatrix}\right) = \begin{bmatrix}20\\50\end{bmatrix}.$$

Find $T\left(\begin{bmatrix}9\\8\end{bmatrix}\right)$. Hint: First, express $\begin{bmatrix}9\\8\end{bmatrix}$ as a LC of $\begin{bmatrix}6\\5\end{bmatrix}$ and $\begin{bmatrix}5\\4\end{bmatrix}$ and use properties of linear transformations on that LC.

9. Let H be the subspace of $\mathcal{F}$ consisting of all functions f in $\mathcal{F}$ whose graph intersects (ie, crosses or "touches") the x-axis in exactly two places. Also, let the "zero function" be a member of H.

Explain why H passes or fails each of the individual parts of the definition for H to be a subspace of $\mathcal{F}$.