1. Find \( \int \frac{x - 4}{x^2 + 6x + 90} \, dx \) by the methods we have discussed in class. Show all your steps.
2. Use integration by parts to find \( \int \arccos x \, dx \). Hint: The derivative of \( \arccos x \) is \( \frac{-1}{\sqrt{1 - x^2}} \). Show all your steps.

3. Suppose values of a function \( r(t) \) have been recorded as follows:

<table>
<thead>
<tr>
<th>time ( t )</th>
<th>1</th>
<th>3</th>
<th>5</th>
<th>7</th>
<th>9</th>
<th>11</th>
<th>13</th>
</tr>
</thead>
<tbody>
<tr>
<td>function ( r(t) )</td>
<td>4</td>
<td>5</td>
<td>7</td>
<td>10</td>
<td>14</td>
<td>19</td>
<td>25</td>
</tr>
</tbody>
</table>

Find each of the following estimates of the integral \( \int_1^{13} r(t) \, dt \) using only the information in the table. If the table doesn’t supply the information needed, explain why not.

TRAP(3) 

MID(3)

SIMP(3)
4. Consider the power series: \( \sum_{n=1}^{\infty} \frac{1 \cdot 3 \cdot 5 \cdot 7 \cdots (2n - 1)}{n!} 3^n(x - 5)^n \)

4A. Explicitly, what is the 5th term of the series? (Write the coefficient as a fraction; don’t cancel any common factors).

4B. Find the interval and radius of convergence for this series.

5A. What are the first five terms of the harmonic series?

5B. What does the harmonic series converge to?

5C. What are the first five terms of the alternating harmonic series?
6. Suppose you begin to take a 50mg dose of a drug every 12 hours. Suppose that the "half life" of this drug in your system is 4 hours; that is, 50% of any given dose is still in your system 4 hours after you take it.

6A. Just after you take the third dose, how many mg of the drug is in your system?

6B. What expression in "closed form" represents how much of this drug is in your system right after the $n^{th}$ dose is taken?

6C. Use the correct answer to 6B to determine how much of the drug is in your system after you've taken one week's worth of doses.

6D. In the long term, how much of the drug is in your system, just after you take a dose?
7. A trough with the following shape and cross-section contains water up to the 3’ mark. Set up (but do not evaluate) the integral which represents how much work is done to empty the trough down to the 1’ mark, if all the water has to be pumped to a height of 5’ above the top of the trough. (Water weighs 62.4 lbs/cu ft).

\[ b^2 + (4-h)^2 = 16 \]

**Answer**

8. Let \( f(x) = \sqrt{x} \).

8A. Find from scratch the first three non-zero terms of the Taylor series for \( f \) in powers of \( x - 16 \).

8B. Use your approximation to estimate \( \sqrt{15} \). Show your answer to 6 digits after the decimal point.

8C. To 6 digits after the decimal point, what does your calculator give for \( \sqrt{15} \)? Also, what is the difference between this value and the approximation from 8B?
9A. Use a substitution and the series you memorized for $e^x$ to find the first four non-zero terms of the Taylor series for $f(t) = 5t^3 e^{2t^2}$.

9B. What is the ninth derivative of $f$, when evaluated at $t = 0$?

10. Solve the initial-value problem: \[ \frac{dy}{dx} = \frac{1}{e^y \sqrt{x}}; \quad y(0) = 1. \]
11. You bake a cake in celebration of the end of math 106. The cake comes out of the oven at 350 degrees, and is left to cool in a room where the air is 70 degrees. The cake is 150 degrees 1/2 hour later. What’s the earliest you can frost it, given the cake can be no warmer that 75 degrees before it can be frosted? (The cake’s temperature obeys Newton’s law of cooling. Solve this problem by setting up the DE and begin your solution from there).
12. Initially, an 8000 gallon tank full of water has 50 pounds of chem-E dissolved in it. At noon, a pipe carrying 10 gallons of water per minute starts dumping into the tank; there are 0.025 lbs/gal of chem-E dissolved in this water. The water in the tank is kept thoroughly mixed and is emptied through another pipe, also at 10 gal/min. Let \( E(t) \) be the amount of chem-E in the tank at any time \( t \).

12A. Find a DE representing the rate of change in the amount of chem-E in the tank.

12B. Solve the DE with the given initial condition.

12C. How many pounds of chem-E are in the tank at 1pm?

12D. How many pounds of chem-E are there in the tank in the long-term?
13. Plutonians (they're from the planet Pluto, of course) survive a crash landing in the Antarctic. Initially there were 1000 Plutionians on board their spaceship. They find themselves in an environment that can sustain 10,000 of them ultimately. Their population as a function of time $t$ in months can be modeled by the logistic equation, with $k = 0.005$.

13A. What is the logistic equation for this problem?

13B. What is its solution? (Give me the “A”, too).

13C. How long before the population reaches 5000?

13D. At the time the population reaches 5000, something important happens to the growth rate...what? Be clear.