

Name: _____

Math 206: Fall 2013
Final Exam

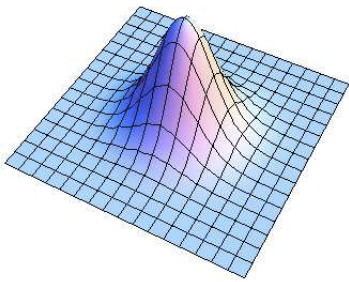
Read directions carefully and show all your work. Partial credit will be assigned based upon the correctness, completeness, and clarity of your answers. Correct answers without proper justification or those that use unapproved short-cut methods will not receive full credit. If you use a calculator to help find an answer, you must write down enough information on what you have done to make your method understandable.

Good Luck!

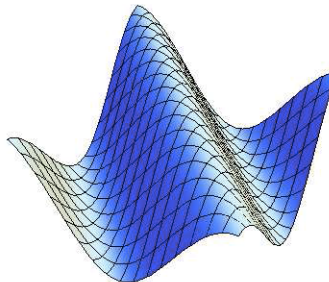
1. (26 points) Solve the following problems. Write your answers on this exam. For all parts of question 1 only, no justification necessary, no partial credit available.

(a) Match each sketch with an equation.

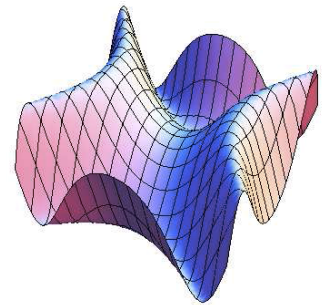
(I)



(II)



(III)



(A) $f(x, y) = e^{-x^2 - y^2}$

(B) $f(x, y) = \cos(x + y)$

(C) $f(x, y) = \sin(x^2 - y^2)$

(D) $f(x, y) = x^4 + y^4$

(b) Let R be the two dimensional region shown in the figure below. What is $\int_R f(x, y) dA$?

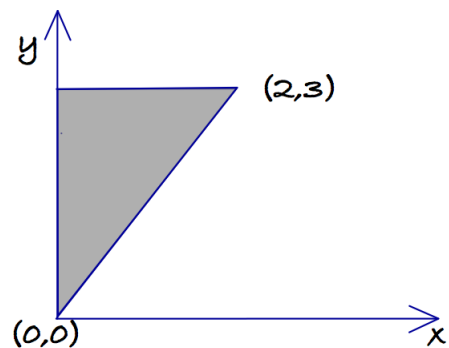
i. $\int_0^3 \int_0^{2y/3} f(x, y) dx dy$

ii. $\int_0^3 \int_0^{3x/2} f(x, y) dx dy$

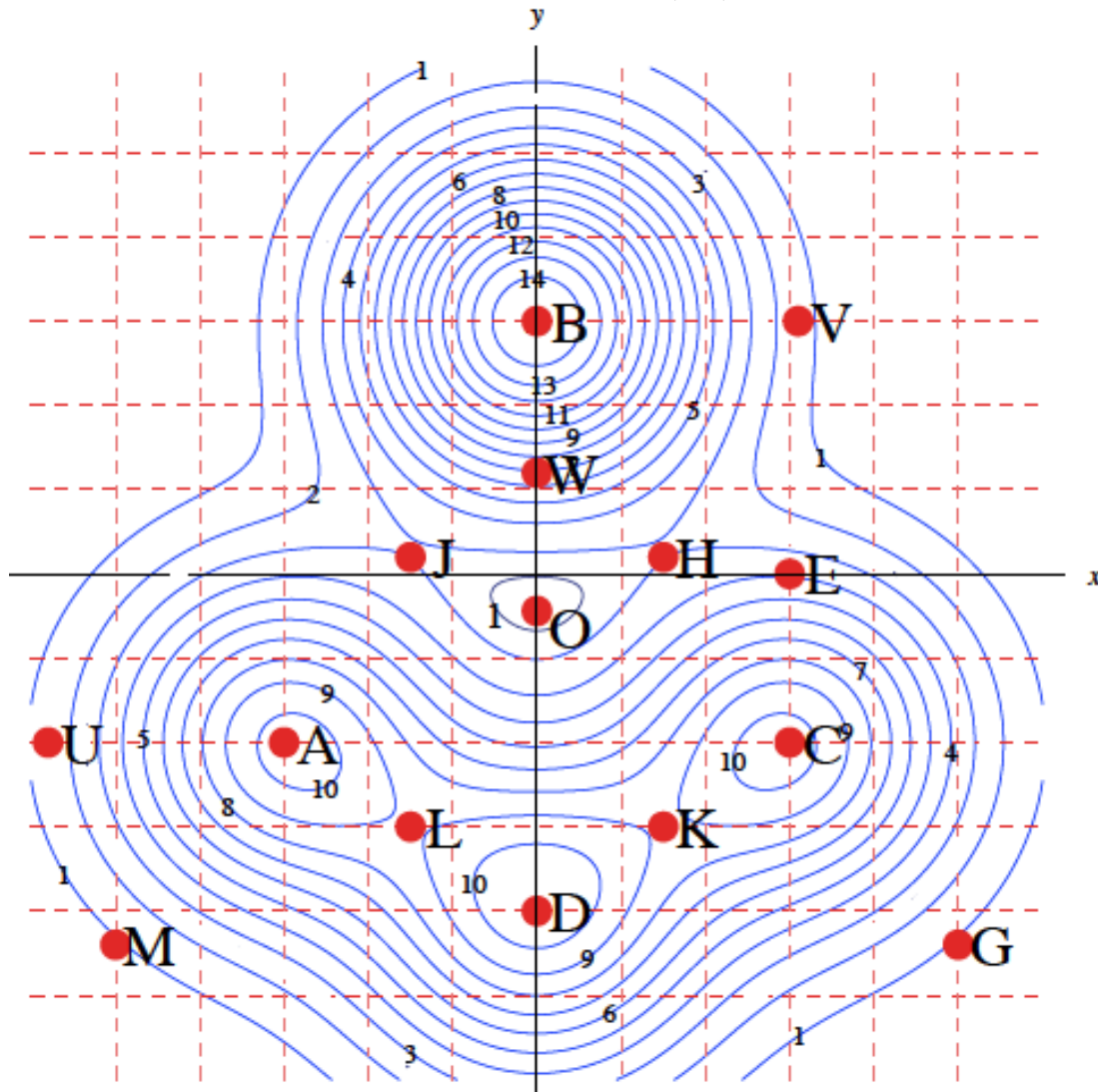
iii. $\int_0^3 \int_0^{2y/3} f(x, y) dy dx$

iv. $\int_0^2 \int_0^{3x/2} f(x, y) dy dx$

v. $\int_0^3 \int_{3x/2}^2 f(x, y) dx dy$



(c) The diagram below depicts contours of a function $f(x, y)$.



Enter one label into each of the boxes.

At which point is $f_x > 0$, $f_y = 0$?

At which point is $f_{\vec{u}} = 0$, and $f_{\vec{v}} < 0$, where $\vec{u} = \frac{1}{\sqrt{2}}(\hat{i} + \hat{j})$ and $\vec{v} = \frac{1}{\sqrt{2}}(\hat{i} - \hat{j})$?

At which point does f have a global maximum?

At which point does f have a local minimum?

At which point is f maximal under the constraint $y = 0$?

At which point is the length of the gradient maximal?

(d) The figure below shows the contour diagram of which function $f(x, y)$?

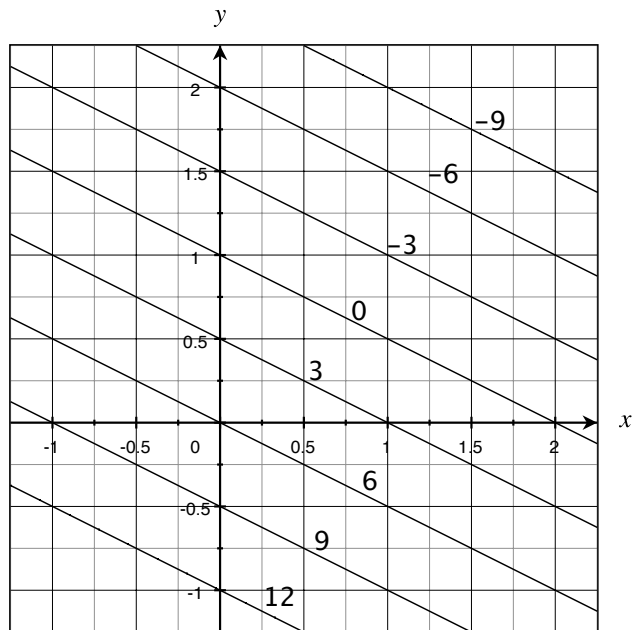
i. $f(x, y) = 6y - 3x + 6$

ii. $f(x, y) = \frac{1-x}{2}$

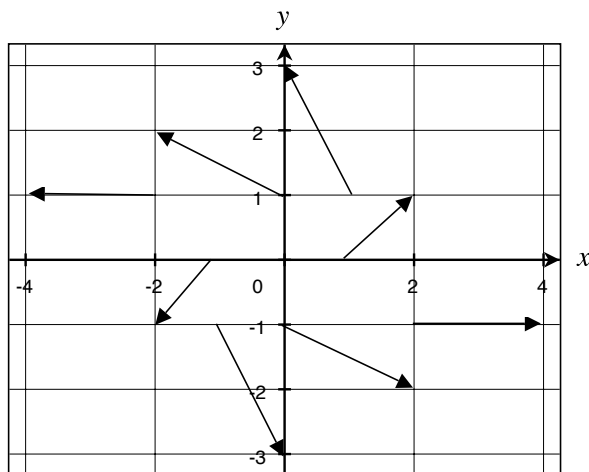
iii. $f(x, y) = e^{-3x-6y+6}$

iv. $f(x, y) = -3x - 6y$

v. None of the above.



(e) A portion of a vector field \vec{F} is shown below. Find $\vec{F}(0, 1)$.



(f) Suppose that $f(x, y)$ has continuous first and second order partial derivatives at $(1, 2)$ and that $f_{xx}(1, 2) = 2$, $f_{yy}(1, 2) = 0$ and $f_{xy}(1, 2) = -1$. Which of the following statements is most likely to be true?

i. $f_y(1, 2) > f_y(1, 2.01)$.

ii. $f_y(1, 2) < f_y(1, 2.01)$.

iii. $f_y(1, 2) = f_y(1, 2.01)$.

iv. It is impossible to determine the relationship between $f_y(1, 2)$ and $f_y(1, 2.01)$ from the given information.

Do the remaining problems in the blue book. Show work for full credit.

- (6 points) Consider the plane $x = 5 + 4y + 2z$ and the line $x = a + bt$, $y = 2 - 2t$, $z = 2 - t$. Find the value of b such that the line is perpendicular to the plane.
- (7 points) A farm costs $f(x, y)$, where x is the number of cows and y is the number of ducks. There are 10 cows and 20 ducks and $f(10, 20) = 1,000,000$. We know that $f_x(x, y) = 2x$ and $f_y(x, y) = y^2$ for all x, y . Use a technique learned in this class to estimate $f(12, 19)$.
- (7 points) An exam question asks students to find the maximum of $f(x, y)$ on the circle $g(x, y) = 29$, and the gradient vectors of f and g at that point. A student gave the following answer: "The maximum value of f (subject to the constraint) occurs at the point $(2, 5)$ with $\nabla f(2, 5) = 4\hat{i} - 10\hat{j}$ and $\nabla g(2, 5) = 2\hat{i} + 5\hat{j}$." Explain why this answer is incorrect.
- (7 points) Let C_1 be the rectangular loop consisting of four line segments: from $(0, 0)$ to $(1, 0)$, then to $(1, 2)$, then to $(0, 2)$, then back to $(0, 0)$. Suppose C_2 is the triangular loop joining $(0, 0)$ to $(1, 0)$, then to $(1, 2)$ then back to $(0, 0)$, and C_3 is another triangular loop joining $(0, 0)$ to $(1, 2)$, then to $(0, 2)$ and then back to $(0, 0)$.

TRUE/FALSE? Justify your answer.

For any vector field \vec{F} defined on the xy -plane

$$\oint_{C_1} \vec{F} d\vec{r} = \oint_{C_2} \vec{F} d\vec{r} + \oint_{C_3} \vec{F} d\vec{r}.$$

- (7 points) Let $H(x, y, z) = \sin(2x + y) + z$. Find the equation of the tangent plane to the level surface $H(x, y, z) = -5$ at the point $(\pi, \pi, -5)$. (You do NOT need to simplify your answer.)
- (14 points) Consider the polynomial $f(x, y) = \frac{5}{2}x^2 - xy + 15x + \frac{1}{75}y^3 - 3y$.
 - Find the critical point(s) of $f(x, y)$.
 - Use the second derivative test to classify, if possible, the critical point(s) you have found. If you cannot use the second derivative test to describe the critical points state that and explain why.
- (10 points) Find $\int_C \vec{F} d\vec{r}$ where $\vec{F} = (x + z)\hat{i} + 3z\hat{j} + 6y\hat{k}$ and C is the line segment from the point $(1, 2, 2)$ to the point $(0, 3, -4)$.
- (14 points) Consider the integral $\int_0^1 \int_{(8y)^{1/3}}^2 \frac{1}{1 + x^4} dx dy$.
 - Interchange the order of integration. Show your work by including a sketch of the region of integration.
 - Evaluate the integral. (Show your work.)
- (2 points) Where will you spend most of your time during break?