FINAL EXAM

Math 205
12/13/11

Name:

Read all of the following information before starting the exam:

- Show all work in the blue book, clearly and in order if you want to get full credit (matrices should be reduced into RREF with calculator and you can just show the output). I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Circle or otherwise indicate your final answers.
- Please keep your written answers brief; be clear and to the point. I will take points off for rambling and for incorrect or irrelevant statements.
- Put your test into your blue book before handing in your exam.
- This test has 7 problems and is worth 100 points, It is your responsibility to make sure that you have all of the pages!
- Good luck!

1. (*14 points*) Given $A = \begin{bmatrix} 1 & 2 \\ k & 4 \end{bmatrix}$, $\vec{b} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$, $\vec{c} = \begin{bmatrix} -1 \\ -2 \end{bmatrix}$.

a. (*3 pts*) Using A and $\vec{b}$ above, write the associated system of linear equations described by the matrix equation $A\vec{x} = \vec{b}$.

b. (*3 pts*) Using A and $\vec{b}$ above, write the associated system vector equation described by the matrix equation $A\vec{x} = \vec{b}$.

c. (*4 pts*) For what k is the equation $A\vec{x} = \vec{b}$ consistent? Explain.

d. (*4 pts*) For what k is the equation $A\vec{x} = \vec{c}$ consistent? Explain.

2. (*15 points*) Given $A = \begin{bmatrix} 3 & 2 & 2 & 2 \\ 2 & 3 & 2 & 2 \\ 2 & 2 & 3 & 2 \\ 2 & 2 & 2 & 3 \end{bmatrix}$.

a. (*3 pts*) Is 1 an eigenvalue of A? If so, find a basis for the eigenspace for $\lambda = 1$. If not, explain.

b. (*3 pts*) Is $\vec{v} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ an eigenvector of A? If so, find the corresponding eigenvalue. If not, explain.

c. (*2 pts*) What is the characteristic equation of A? (Hint: you shouldn't have to do any calculation.)

d. (*3 pts*) If possible, diagonalize A. (Ie. Find P and D such that $A = PDP^{-1}$).

e. (*4 pts*) What is the Col(A)? What is the Nul(A)? How do you know without calculation? Give a good explanation.

3. (*13 points*) Determine which of the following are subspaces of the appropriate vectorspaces. If the set is a subspace, then find a basis and state the dimension of the subspace. If not, explain why.

a. (*4 pts*) W ={polynomials of the form $a + bt^2$, where $a, b \in \mathbb{R}$}.

b. (*4 pts*) $W = \{$all 2×2 invertible matrices $\}$

c. (*5 pts*) $W = \left\{ \begin{bmatrix} r+t+u \\ s+t-u \\ 2r-s+t+3u \end{bmatrix} \mid r, s, t, u \in \mathbb{R} \right\}$

4. (*20 points*) Consider $\vec{u} = \begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$, $\vec{y} = \begin{bmatrix} -2 \\ 3 \\ 2 \end{bmatrix}$. Let $W = \text{span}\{\vec{u}, \vec{v}\}$.

a. (*3 pts*) Determine the distance between $\vec{u}$ and $\vec{v}$.

b. (*3 pts*) What is the unit vector associated with $\vec{v}$?

c. (*2 pts*) Determine if $\{\vec{u}, \vec{v}\}$ is an orthogonal set.

d. (*4 pts*) Is $\vec{y} \in W$?

e. (*4 pts*) What is the closest vector to $\vec{y}$ in W?

f. (*4 pts*) Find a vector in $W^{\perp}$.

5. (*10 points*)

a. (*6 pts*) Find the least-squares solutions of $A\vec{x} = \vec{b}$ for $\begin{bmatrix} 1 & 5 \\ 3 & 1 \\ -2 & 4 \end{bmatrix}$, $\vec{b} = \begin{bmatrix} 4 \\ -2 \\ -3 \end{bmatrix}$.

b. (*4 pts*) What is the vector in $Col(A)$ that is closest to $\vec{b}$. (Hint: this should be a very quick calculation.)

6. (*12 points*) **a.** (*8 pts*) If possible, orthogonally diagonalize the matrix A. The eigenvalues are $\lambda = 0$ and $\lambda = 5$. $A = \begin{bmatrix} 4 & -2 \\ -2 & 1 \end{bmatrix}$

b. (*4 pts*) Suppose A is a symmetric $n \times n$ matrix. Let B be any $n \times n$ matrix. Is BAB^T orthogonally diagonalizable? Why or why not?

7. (*16 points*) The canonical basis for real valued 2×2 matrices ($M_{2\times2}$) is β. Where $\beta = \left\{ E_{11} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, E_{12} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, E_{21} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, E_{22} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$. Let T be a linear transformation that acts on 2×2 matrices, $T(B) = \frac{B+B^T}{2}$.

a. (*4 pts*) What is the kernel of T?

b. (*4 pts*) Find $E1 = T(E_{11})$, $E2 = T(E_{12})$, $E3 = T(E_{21})$, and $E4 = T(E_{22})$.

c. (*4 pts*) Consider the coordinate vector for X, $[X]_\beta$. Here $X \in M_{2\times2}$ and $[X]_\beta \in \mathbb{R}^4$. Determine the coordinate vectors for $E1$, $E2$, $E3$, and $E4$ using the basis β. (Ie. Find $[E1]_\beta$, etc)

d. (*4 pts*) Recall a linear transformation T can be represented as a matrix transformation. Determine the matrix representation, A, for T with respect to the basis β. (Hint: A is a 4×4 matrix.)

8. (*2 points*) BONUS: Which of the following linear algebra jokes is your favorite?

1. What do you call a young eigensheep?
A lamb, duh!!!

2. My mentor insisted I do a presentation on the Gram-Schmidt process. It went horribly! The lighting was so dim people could barely see. All my slides were horribly smudged, and the fonts were typeset way too small. My mentor was furious.
"I'm sorry sir!" I wailed. "I'm terrible at projections!"

3.