

NAME:

Math 105C&D - Final Exam - December 13, 2007

Instructions: Show all of your work and circle your final answers. Your work should flow logically and be easy to follow. Cross out any work that you do not want considered. Calculators are allowed, but notes and books are not.

Problem	Max	Score
1	10	
2	10	
3	10	
4	10	
5	12	
6	9	
7	12	
8	12	
9	15	
TOTAL	100	

1. (10 points) Is $F(x) = \frac{x^2}{2} \ln x - \frac{x^2}{4} + 1$ an antiderivative of $f(x) = x \ln x$? Justify your answer.

2. (10 points) Evaluate $\lim_{x \rightarrow 0} \frac{e^{x^2} - 1}{1 - \cos x}$. (Note: e^{x^2} is $e^{(x^2)}$, not $(e^x)^2$.)

3. (10 points) Suppose $f(x) = 2x - 3x^2$. Use the limit definition of the derivative to find $f'(x)$.

4. (10 points) Use the Fundamental Theorem of Calculus to evaluate $\int_1^2 \left(\frac{x}{2} + \frac{2}{x} \right) dx$. (Your final answer should be *exact*, rather than a decimal approximation.)

5. (12 points) Suppose we know that $\int_4^{10} f(x) dx = 5$; $\int_4^7 f(x) dx = -2$; and $\int_{10}^{13} f(x) dx = 1$, for some function $f(x)$.

(a) Find $\int_7^{10} f(x) dx$.

(b) Find $\int_4^7 (f(x) + 2) dx$.

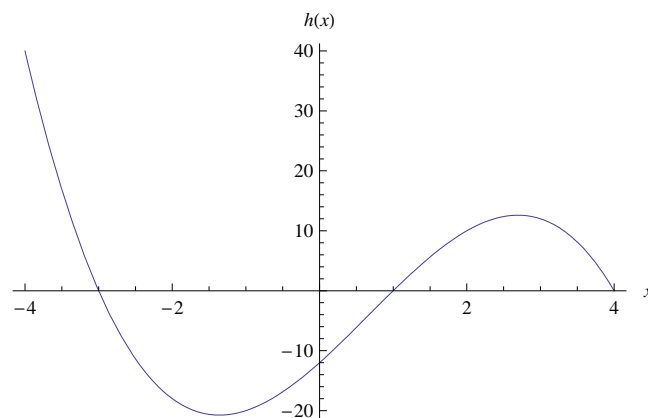
(c) Find $\int_7^{10} f(x - 3) dx$. (Hint: How are the graphs of $f(x)$ and $f(x - 3)$ related?)

6. (9 points) Determine whether the following statements are true or false. No work is needed. For these statements, $f(x)$ can be any function such that $f(x)$, $f'(x)$, and $f''(x)$ are all continuous on $(-\infty, \infty)$.

(a) If $f'(2) = 0$ and $f''(2) > 0$, then $f(x)$ must have a local minimum at $x = 2$. True / False

(b) If $f''(3) = 0$, then $f(x)$ must have an inflection point at $x = 3$. True / False

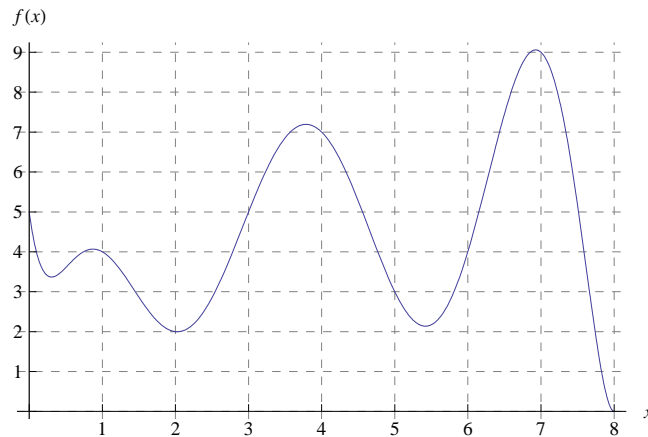
(c) For the function $h(x)$ given below, $\int_{-1}^{-2} h(x) dx < 0$. True / False



7. (12 points) A spotlight is placed on the ground 30 feet in front of the wall of a building. The spotlight shines on a 2-foot tall smurf that is between the light and the wall, producing a shadow on the wall. If the smurf walks from the wall toward the spotlight at a rate of 5 feet per second, at what rate is the tip of the smurf's shadow moving up the wall when the smurf is 10 feet away from the spotlight?

(To receive full credit for this problem, draw a picture, label all relevant quantities, explain what your variables are, and include units in your final answer.)

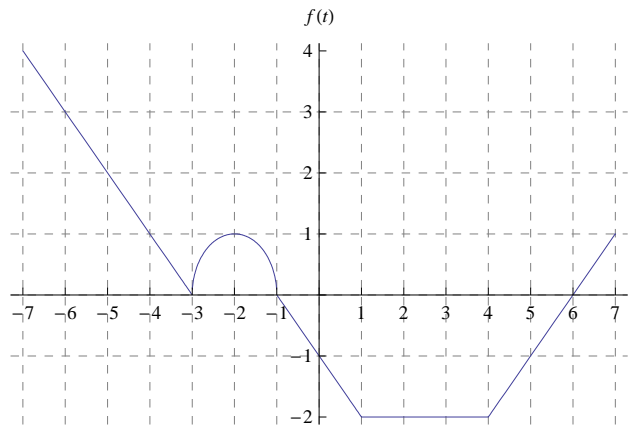
8. (12 points) Use the graph of $y = f(x)$, given below, to answer question (a).



(a) Use a midpoint sum on three intervals to approximate $\int_1^7 f(x) dx$. Your final answer should be a whole number or a fraction (in reduced form).

(b) Now, suppose we want to approximate $\int_0^5 \arctan(x) dx$. (This has nothing to do with the function $f(x)$ above). Use summation notation (Σ -notation) to write out R_{20} , a right endpoint sum on 20 subintervals. (Leave your answer in summation notation - do not evaluate it.)

9. (15 points) The graph of $f(t)$, which is made using straight lines and a semicircle, is given below. Let $A(x) = \int_{-2}^x f(t) dt$. (In class, our notation for this was $A_{f,-2}(x)$.)



- (a) Find and classify all local extrema of $A(x)$. Give a brief explanation.
- (b) On what interval(s) in $[-7, 7]$ is $A(x)$ concave up? Give a brief explanation.
- (c) Find a formula for $A(x)$ for the case where $1 \leq x \leq 4$.