Math 206 Final Exam

Name: ____________________

- You have two hours.
- Show all your work.

1. (10 marks) Find the equation of the plane tangent to the graph of

$$f(x, y) = \frac{1}{x^2 + y^2 + 1}$$

at the point $(1, 0)$.

2. (10 marks) $f(x, y) : R^2 \to R$ and $(x, y) = \vec{g}(r, \theta) = (r\cos\theta, r\sin\theta)$. Use the chain rule to find an expression for

$$\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y}.$$

3. (10 marks) $f(x, y, z) = \ln(x^2 + y^2 + z^2)$

a. Calculate the directional derivative of $f(x, y, z)$ at the point $(1, 0, 0)$ in the direction $(0, 2, 0)$.

b. Find the equation of the plane tangent to the level surface $f(x, y, z) = 0$ at the point $(1, 0, 0)$.

4. (10 marks) Suppose $f(x, y, z) = x^2 + y^2 + z^2$.

a. Evaluate $\mathrm{div}(\vec{\nabla} f)$.

b. Evaluate $\mathrm{curl}(\vec{\nabla} f)$.

5. (10 marks) C is the helix parametrized by $\vec{p}(t) = (\cos t, \sin t, t)$, $0 \leq t < 2\pi$. Answer 2 of the following problems.

a. Find the length of C.

b. Evaluate the path integral of $f(x, y, z) = x^2 + y^2 + z^2$ along C.

c. Evaluate the path integral of $\vec{F}(x, y, z) = (-y, x, z)$ along C.

6. (10 marks) M is the torus obtained by revolving the circle of radius one (centred at $(2, 0)$ in the xz-plane) about the z-axis and with outward pointing normal. Answer 2 of the following problems.

a. Find the surface area of M.

b. Evaluate the surface integral of $f(x, y, z) = x^2 + y^2 + z^2$ over M.

c. Evaluate the surface integral of $\vec{F}(x, y, z) = (-y, x, z)$ over M.

6. continued.

7. (10 marks) C is the helix parametrized by $\vec{p}(t) = (2\pi \cos t, 2\pi \sin t, \frac{t}{2\pi})$, $0 \leq t < 2\pi$. Use the Fundamental Theorem for path integrals to evaluate the path integral of

$$\vec{F}(x, y, z) = (-e^y \sin x, e^y \cos x, 1)$$

along C.

8. (10 marks) C is the unit circle centred at the origin and oriented counter-clockwise. Use Green's Theorem to evaluate the path integral of

$$\vec{F}(x, y) = (\sqrt{x^2 + (y-2)^2}, \sqrt{x^2 + (y-2)^2})$$

along C.

9. (10 marks) S is the unit sphere centred at the origin with outward pointing normal. Use Gauss' Theorem to evaluate the surface integral of

$$\vec{F}(x, y, z) = (0, (x^2 + y^2 + z^2)^{\frac{3}{2}}, 0)$$

over S.

10. (10 marks) M is that part of the surface $z = \frac{1}{1+x^2+y^2}$ contained in the cylinder $x^2 + y^2 = 1$ with orientation in the direction of the positive z axis. Use Stokes' Theorem to evaluate the surface integral of curl$\vec{F}$ over M where

$$\vec{F}(x, y, z) = (-y, x, z)$$