NAME:

Instruction: Read each question carefully. Explain ALL your work and give reasons to support your answers.
Advice: DON'T spend too much time on a single problem.

<table>
<thead>
<tr>
<th>Problems</th>
<th>Maximum Score</th>
<th>Your Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
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1. Let $f : \mathbb{R}^2 \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}^3$ be given by

$$f(x, y) = 3x^3 + y^2 - 9x + 4y \quad \text{and} \quad g(t) = \left(\frac{t^2}{6}, e^{t/3}, 1 - t\right).$$

(8 pts) (i) Classify all critical points of $f$ (local max/min, saddle points, etc.).

(7 pts.) (ii) Find the Jacobian matrix $J(g \circ f)(0, 1)$ of $g \circ f$ at $(0, 1)$. 
2. (10 pts) (i) Use change of variables to evaluate the following double integral. [Hint: First determine the region of integration.]

\[ \int_{-1}^{1} \int_{0}^{\sqrt{1-y^2}} 2xy^2 \, dx \, dy \]

(10 pts) (ii) Find the double integral \( \int \int_R x \, dA \) where \( R \) is the region bounded by the curve \( y = x^3 \) and the line \( y = x \). [Hint: There are two parts of the integral.]
3. Consider the following function \( F : \mathbb{R}^3 \to \mathbb{R} \) given by
\[
F(x, y, z) = xy + z^2.
\]

(5 pts) (i) Find the directional derivative \( D_u F(1, 1, 1) \) of \( F \) at the point \((1, 1, 1)\) in the direction of \( u = 2i - j + 2k \).

(5 pts) (ii) Find a direction (give a unit vector) in which \( F \) decreases most rapidly at the point \((1, 1, 1)\).

(5 pts) (iii) Find an equation of the plane tangent to the level surface \( F(x, y, z) = 2 \) at the point \((1, 1, 1)\).
4. (8 pts) (i) Let $C$ be the path formed by the triangle with vertices $(0,0), (1,0)$, and $(0,1)$, oriented counterclockwise. Use Green’s theorem to evaluate the line integral
\[ \int_C 2xy \, dx + (x + 1)^2 \, dy. \]

(12 pts) (ii) Let $F(x, y) = (-e^{-x} \ln y, \frac{e^{-x}}{y})$. Determine whether the vector field $F$ is path independent. If so, find a function $f$ so that $\nabla f = F$. 
5. Consider the curve $C$ given by the parametrization

$$\mathbf{x}(t) = (t, 1 - t, t^2) \quad 1 \leq t \leq 2.$$ 

(8 pts) (i) Find the work done by the force $F(x, y, z) = (xy, z, x + y)$ along the curve $C$.

(7 pts.) (ii) A wall $W$ is to be built on top of the curve $C$ where the height is given by $f(x, y, z) = x$. Find the surface area of the wall $W$. 
6. Let $F(x, y, z) = (x^2y, 2xz, yz^3)$.

(5 pts) (i) Find the divergence $\text{div } F$ of $F$.

(10 pts) (ii) Use Gauss’ (or Divergence) theorem to evaluate the flux of the vector field $F$

$$\oiint_{\partial S} F \cdot \mathbf{n} \, d\sigma$$

where $\partial S$ is the surface of the rectangular box $S$ determined by

$$0 \leq x \leq 1, 0 \leq y \leq 2, 0 \leq z \leq 3.$$
7. Let \( F(x, y, z) = (z, x, y) \). Suppose \( M \) is the portion of the paraboloid \( z = x^2 + y^2 + 2 \) that lies inside the solid cylinder \( x^2 + y^2 \leq 1 \).

(i) Find \( \text{curl } F \).

(ii) Write a parametrization for the surface \( M \). Be sure to indicate the domains for the parameters.

(iii) Use Stokes' theorem to evaluate the path integral
\[
\oint_{\partial M} F \cdot dx
\]
[Hint: Use parts (i) and (ii)].