MATH206A MULTIVARIABLE CALCULUS - PROF. P. WONG

FINAL EXAM - DECEMBER 12, 2007

NAME:

Instruction: Read each question carefully. Explain **ALL** your work and give reasons to support your answers.

Advice: DON'T spend too much time on a single problem.

Problems	**Maximum Score**	**Your Score**
1.	15	
2.	20	
3.	15	
4.	20	
5.	15	
6.	15	
7.	20	
Total	120	

1. Let $f : \mathbb{R}^2 \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}^3$ be given by

$$f(x, y) = 3x^3 + y^2 - 9x + 4y \quad \text{and} \quad g(t) = (\frac{t^2}{6}, e^{t/3}, 1 - t).$$

(8 pts) (i) Classify all critical points of f (local max/min, saddle points, etc.).

(7 pts.) (ii) Find the Jacobian matrix $J(g \circ f)(0, 1)$ of $g \circ f$ at $(0, 1)$.

2. (10 pts) (i) Use change of variables to evaluate the following double integral. [Hint: First determine the region of integration.]

$$\int_{-1}^{1}\int_{0}^{\sqrt{1-y^2}} 2xy^2 \; dxdy$$

(10 pts) (ii) Find the double integral $\int\int_R x \; dA$ where R is the region bounded by the curve $y = x^3$ and the line $y = x$. [Hint: There are two parts of the integral.]

3. Consider the following function $F : \mathbb{R}^3 \to \mathbb{R}$ given by

$$F(x, y, z) = xy + z^2.$$

(5 pts) (i) Find the directional derivative $D_{\mathbf{u}}F(1, 1, 1)$ of F at the point $(1, 1, 1)$ in the direction of $\mathbf{u} = 2\mathbf{i} - \mathbf{j} + 2\mathbf{k}$.

(5 pts) (ii) Find a direction (give a unit vector) in which F decreases most rapidly at the point $(1, 1, 1)$.

(5 pts) (iii) Find an equation of the plane tangent to the level surface $F(x, y, z) = 2$ at the point $(1, 1, 1)$.

4. (8 pts) (i) Let C be the path formed by the triangle with vertices $(0,0),(1,0))$, and $(0,1)$, oriented counterclockwise. Use Green's theorem to evaluate the line integral

$$\oint_C 2xy\ dx + (x+1)^2\ dy.$$

(12 pts) (ii) Let $F(x,y) = (-e^{-x}\ln y, \frac{e^{-x}}{y})$. Determine whether the vector field F is path independent. If so, find a function f so that $\nabla f = F$.

5. Consider the curve C given by the parametrization

$$\mathbf{x}(t) = (t, 1 - t, t^2) \qquad 1 \leq t \leq 2.$$

(8 pts) (i) Find the work done by the force $F(x, y, z) = (xy, z, x + y)$ along the curve C.

(7 pts.) (ii) A wall W is to be built on top of the curve C where the height is given by $f(x, y, z) = x$. Find the surface area of the wall W.

6. Let $F(x, y, z) = (x^2y, 2xz, yz^3)$.

(5 pts) (i) Find the divergence div F of F.

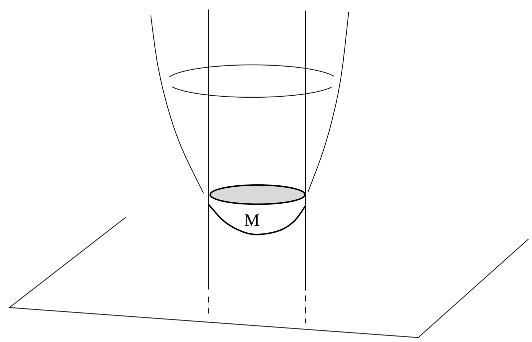
(10 pts) (ii) Use Gauss' (or Divergence) theorem to evaluate the flux of the vector field F

$$\oiint_{\partial S} F \cdot \mathbf{n} \, d\sigma$$

where ∂S is the surface of the rectangular box S determined by

$$0 \leq x \leq 1, 0 \leq y \leq 2, 0 \leq z \leq 3.$$

7. Let $F(x, y, z) = (z, x, y)$. Suppose M is the portion of the paraboloid $z = x^2 + y^2 + 2$ that lies inside the solid cylinder $x^2 + y^2 \leq 1$.



(5 pts.)(i) Find curl F.

(5 pts.)(ii) Write a parametrization for the surface M. Be sure to indicate the domains for the parameters.

(10 pts.)(iii) Use Stokes' theorem to evaluate the path integral

$$\oint_{\partial M} F \cdot d\mathbf{x} \quad \text{[Hint: Use parts (i) and (ii).]}.$$