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I ___ II ___ III ___ IV ___ V ___ VI ___ VII ___ VIII ___ IX ___ X ___

XI ___ XII ___ XIII ___ XIV ___ XV ___ TOTAL ___

December 12
2007

Mathematics 106a
Calculus II
Final Examination

Mr. Haines

(5) I. What is the value of $\frac{100}{\sqrt{2\pi}} \int_{-\infty}^0 e^{-\frac{x^2}{2}} dx$?

(5) II. Write the fraction $\frac{1}{x(x^2 + 1)}$ as the sum of two rational functions.

(5) III. If $\int_1^{\infty} \frac{1}{x^p} dx$ converges to $2/3$, what is the value of p ? Justify your answer.

(10) IV. Find the integrals:

A. $\int x(2 + \cos x)dx$.

B. $\int \frac{3x^2 + 1}{x(x^2 + 1)} dx$.

(10) V. For $f(x) = \sqrt[3]{x}$

A. Give the second degree Taylor polynomial for f at $x_0 = 8$.

B. Use this polynomial to estimate $\sqrt[3]{9}$.

C. What is the largest possible error that could have occurred in your estimate in part B?

Recall that if you use the Taylor polynomial of degree n at x_0 to approximate

$f(x)$ for x in an interval I containing x_0 then $\frac{K_{n+1} |x - x_0|^{n+1}}{(n+1)!}$ is an upper

bound for the approximation error. [K_{n+1} is an upper bound for the absolute value of the $(n+1)$ st derivative of f on I .]

(5) VI. Starting with the Maclaurin series for $\frac{1}{1+x}$,

A. find a power series expression for $\int \frac{1}{1+x^3} dx$.

B. For what values of x does this formula hold?

(10) VII. A spherical tank of radius 10 feet is buried 8 feet below ground and is filled to a height of 13 feet from the bottom of the tank with gasoline (42 pounds per cubic foot). Write an integral equal to the work done in pumping all the gasoline to ground level.

(5) VIII. Consider the region bounded by $y = 0$, $x = 2$, and $y = x^2$. Write an integral equal to the volume of object created when the region is revolved about the x -axis.

(10) IX. Find the solution that passes through (0,2) for the equations:

A. $\frac{dy}{dx} = y \cos x$

B. $\frac{dy}{dx} = x \cos x$

(5) X. Let $I = \int_0^2 x^2 dx$.

A. Use the Fundamental Theorem of Calculus to evaluate I exactly.

B. Compute the approximating sum L_4

C. Compute the approximation error $|I - L_4|$

(5) XI. Write (but do not evaluate) an integral that gives the arc length of the graph of $y = e^{2x}$ over the interval $[1, 5]$.

(10) XII. Do these integrals converge? Justify your answers.

$$\text{A. } \int_2^{\infty} \frac{1}{x - \sqrt{x}} dx$$

$$\text{B. } \int_1^{\infty} \frac{\cos x}{x^2} dx$$

(5) XIII. What is the sum of the series $\sum_{k=1}^{\infty} \left(\frac{2}{3}\right)^k$?

(5) XIV. Test to determine whether the series $\sum_{n=1}^{\infty} \frac{n^3}{3^n}$ converges or diverges and explain why.

(5) XV. For the series $\sum_{n=1}^{\infty} (-1)^{n+1} \left(\frac{x-2}{3}\right)^n$

A. Give the radius of convergence:

B. Give the interval of convergence: