

MATH 105, Sections A&B

Final Exam

Tuesday 12/12/06

1. (20 points) Find the domain of the following function.

$$f(x) = \sqrt[4]{\frac{(1-x)(5-x^2)}{x^6}}$$

2. (20 points) Let $f(x) = x^4 - 4x^3$.

- Find local maxima of $f(x)$.
- Find local minima of $f(x)$.
- Find inflection points of $f(x)$.

3. (20 points) Let $f(x) = \begin{cases} ax^4 + 80 & \text{if } x > 2 \\ bx^2 + 4a & \text{if } x \leq 2 \end{cases}$. Find the values of a and b such that $f(x)$ is continuous and differentiable at $x = 2$.

4. (20 points) An object moves along a straight line such that its acceleration is $a(t) = 15\sqrt{t} + \frac{3}{\sqrt{t}}$. The experiment shows that at $t = 1$ we have the following data: the velocity of the object is 19 units per second, and it's 9 units far from the origin in the positive direction.

- Find the velocity at time $t = 4$.
- Find a function that describes the position of the object at time t .

5. (20 points) A mold grows at a rate proportional to the amount present. Initially, its weight was 10 grams; after 3 days, it was $10e$ grams.

- Determine a differential equation that describes the process and a function that expresses the weight of the mold as a function of time.
- What is the weight of the mold after 15 days?
- After how many days it will be 10 times the initial size?
- Use the differential equation to determine how fast the mold is growing when its weight is 24 grams?

6. (20 points) Simplify each of the following expressions as much as possible, i.e., rewrite each of them without using trigonometric or inverse trigonometric functions at all.

a) $\cos\left(\arctan \frac{2}{\sqrt{x}}\right)$ b) $\cos\left(2 \arcsin \frac{2}{\sqrt{x}}\right)$ c) $\sin^2(\arctan \sqrt{2}) + \cos^2(\arctan \sqrt{2})$

7. (20 points) A wire of length 2 meters is to be cut into two pieces. One piece will be used to form a square; the other, to form a circle.

- What should be the side of the square to maximize the sum of the areas of the pieces?
- What should be the side of the square to minimize the sum of the areas of the pieces?

8. (20 points) Evaluate the following limits. If you get $+\infty$ or $-\infty$, state so explicitly.

a) $\lim_{x \rightarrow +\infty} \frac{x^2}{\ln x}$ b) $\lim_{x \rightarrow 1^+} \frac{x^2}{\ln x}$ c) $\lim_{x \rightarrow 1^-} \frac{x^2}{\ln x}$ d) $\lim_{x \rightarrow 0^+} \frac{x^2}{\ln x}$ e) $\lim_{x \rightarrow -\infty} \frac{x^2}{\ln |x|}$

9. (20 points) The length of the base of a right triangle is increasing at the rate of 2 inches per minute. At the same time, the height of the triangle is decreasing in such a way that the length of the hypotenuse remains 10 inches. Suppose that at a certain instant t_0 the length of the base is 6 inches.

- a) How quickly is the height of the triangle changing at t_0 ?
b) How quickly is the area of the triangle changing at t_0 ?

10. (20 points) Find real numbers A and B such that

$$A \leq \int_{\pi/4}^{5\pi/4} e^{\sin^2 x} dx \leq B.$$

11. (20 points) Find the area of the region in the xy -plane bounded by the curves $y = e^{2x}$, $y = e^x$, and $x = \ln 2$.