I. (20) Evaluate

A. \(\int x^2 \sin(x^3) \, dx\)

B. \(\int \frac{x}{x^2 + 1} \, dx\)
C. \[ \int_0^{\ln 2} xe^x\,dx \]

D. \[ \int \tan^2 x \sec^2 x\,dx \]
II. (5) Suppose $f$ is a function which is non-negative, increasing, and concave down on the interval $[0, 1]$. Let $I = \int_{0}^{1} f(x) \, dx$. Suppose you use $L_{12}$, $R_{12}$, $T_{12}$, and $M_{12}$, to estimate $I$.

Fill in the blanks below with the symbols $I$, $L_{12}$, $R_{12}$, $T_{12}$, and $M_{12}$ in the correct order:

$$\_\_\_\_\_ \leq \_\_\_\_\_ \leq \_\_\_\_\_ \leq \_\_\_\_\_ \leq \_\_\_\_\_$$

III. (5) Use Euler’s method with three equal steps on the differential equation $\frac{dy}{dt} = y + t$ to estimate $y(4)$ if $y(1) = 2$. 

IV. (10) Set up, but do not evaluate,

A. the integral which gives the arc length of the graph of $y = x^2$ over the interval $[0, 1]$.

B. the integral which gives the volume of the solid obtained when you revolve about the $x$-axis the area under $y = \frac{1}{\sqrt{x} + 1}$ that lies between $x = 1$ and $x = 2$. 
V. (5) A 500-foot rope that weighs 2 pounds per foot is hanging from the roof of a 2000-foot building. Set up an integral whose value is the amount of work required to pull the rope up to the top of the building? Do not evaluate the integral.

VI. (5) In the year 2035 you will receive $10,000 per year for 20 years (until 2055). What is the present value of this income stream, assuming an annual interest rate of 5% compounded continuously?
VII. (5) Give the fourth order Taylor polynomial for \( f(x) = \frac{1}{1-x} \) centered at \( x = 0 \).

VIII. (5) As you know, the Taylor polynomial \( p_3(x) = x - \frac{x^3}{6} + \frac{x^5}{120} \) can be used to approximate \( \sin x \) for values of \( x \) near 0. The error in this approximation is bounded by \( \frac{K_6 |x|^6}{6!} \), where \( K_6 \) is a bound on the derivative of \( \sin x \). For what values of \( x \) can we safely use \( p_3(x) \) to approximate \( \sin x \) if we want to be accurate to within \( \frac{1}{500} \)?
IX. (10) Evaluate

A. \( \int_{0}^{\infty} \left( \frac{1}{e^x} \right)^5 \, dx \)

B. \( \lim_{k \to \infty} \frac{k + 1}{3k + 2} \)
X. (15) Determine whether or not each of these series converges, explaining why or why not.

A. \[ \sum_{k=1}^{\infty} \frac{k + 1}{3k + 2} \]

B. \[ \sum_{n=1}^{\infty} \frac{5^n}{n!} \]

C. \[ \sum_{k=1}^{\infty} \left( \frac{\sin k}{k} \right)^4 \]
XI. (10) For each of these power series answer two questions:

1. To what function does it converge?
2. For what values of $x$ does it converge?

A. $1 + \left( \frac{x}{3} \right) + \left( \frac{x}{3} \right)^2 + \left( \frac{x}{3} \right)^3 + \left( \frac{x}{3} \right)^4 + \left( \frac{x}{3} \right)^5 + ...$

B. $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + ...$

XII. (5) As you know, the alternating series $\sum_{k=1}^{\infty} \frac{(-1)^k}{k}$ converges. Does it converge conditionally or absolutely? Explain your answer.