

Math 206 — Final Exam

December 11, 2012

Name: _____

Instructor: _____ Section: _____

1. **Do not open this exam until you are told to do so.**
2. This exam has 9 pages including this cover AND IS DOUBLE SIDED. There are 11 problems. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
3. Do not separate the pages of this exam. If they do become separated, write your name on every page and point this out when you hand in the exam.
4. Please read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret mathematical questions.
5. Show an appropriate amount of work (including appropriate explanation). Include units in your answer where that is appropriate. Time is of course a consideration, but do not provide no work except when specified.
6. You may use any previously permitted calculator. However, you must state when you use it.
7. If you use graphs or tables to find an answer, be sure to include an explanation and sketch of the graph that you use.
8. **Turn off all cell phones and pagers**, and remove all headphones and hats.
9. Remember that this is a chance to show what you've learned, and that the questions are just prompts.

Problem	Points	Score
1	09	
2	06	
3	13	
4	10	
5	12	
6	15	
7	10	
8	06	
9	10	
10	07	
11	02	
Total	100	

Some formulae:

$$\begin{aligned}\nabla \times F &= \left(\frac{dF_3}{dy} - \frac{dF_2}{dz}, \frac{dF_1}{dz} - \frac{dF_3}{dx}, \frac{dF_2}{dx} - \frac{dF_1}{dy} \right) \\ (a_1, a_2, a_3) \times (b_1, b_2, b_3) &= (a_2b_3 - a_3b_2, a_3b_1 - a_1b_3, a_1b_2 - a_2b_1) \\ \cos(2\theta) &= \cos^2(\theta) - \sin^2(\theta) \\ \sin(2\theta) &= 2 \sin(\theta) \cos(\theta) \\ \cos^2(\theta) + \sin^2(\theta) &= 1\end{aligned}$$

1. [9 points] Let D be the region in the xy -plane which is bounded by $x^2 - y^2 = 9$, $x^2 - y^2 = 5$, $xy = 2$, and $xy = 1$. Compute

$$\int \int_D xy(x^2 + y^2) dx dy.$$

2. [6 points] Fill in the bounds of the integral:

$$\int_0^1 \int_0^x f(x, y) dy dx + \int_1^2 \int_0^1 f(x, y) dy dx + \int_2^3 \int_{x-2}^1 f(x, y) dy dx = \int_a^b \int_c^d f(x, y) dx dy.$$

$$a =$$

$$b =$$

$$c =$$

$$d =$$

3. [13 points] In this problem, let

$$F(x, y, z) = \left(e^{x^3}, \frac{3z}{y^2 + z^2}, -\frac{3y}{y^2 + z^2} \right),$$

and let C be the curve parametrized by $p(t) = (\cos(t), \sin(2t), \cos(2t))$ with $0 \leq t \leq 2\pi$.

a. [5 points] Set up (but do not compute) the arc length integral for the curve C ?

b. [8 points] Compute

$$\int_C F \cdot dL.$$

4. [10 points] Let $f(x)$ and $g(y)$ be functions of one variable with continuous second derivatives. Let $u(x, y) = f(x + g(y))$. Compute

$$u_x u_{xy} - u_y u_{xx} = \frac{du}{dx} \frac{d^2u}{dx dy} - \frac{du}{dy} \frac{d^2u}{dx^2}.$$

5. [12 points] Given a sheet of metal lying in the xy -plane with corners at $(7, 0)$, $(6, 0)$, $(3, -4)$, $(5, -8)$, and $(7, -3)$ and mass density x kg/m. What is the total mass of this irregularly shaped metal?

6. [15 points] Let S be the surface parametrized by $p(s, t) = (4s \cos(t), 4s \sin(t), 4s^2)$, with $0 \leq s \leq 1$ and $0 \leq t \leq 2\pi$, and let C be the boundary of S . Also let $F(x, y, z) = (yz e^{xyz}, xz e^{xyz} - z, xy e^{xyz} - y + x)$.

a. [5 points] What is $\nabla \times F$? Is F a conservative vector field?

b. [5 points] What is the parametrization of C ?

c. [5 points] Use Stokes Theorem to compute $\int \int_S \nabla \times F dA$ by finding another surface that also has C as its boundary.

7. [10 points] True or False (no partial credit)

a. [2 points] It is true for any $a, b, c \in \mathbb{R}^3$ we have that $((c \times a) \cdot c) + ((c \times b) \cdot c) = 0$?

b. [2 points] $x^2 + y^2 - 2y = z$ is the equation of a cone.

c. [2 points] $p(t) = (t - 1, 2t - 3, 5t)$ is the equation of a line through the point $(1, 1, 15)$.

d. [2 points] $\tan(\theta) = -1$ in polar coordinates defines the same curve as $x + y = 0$ in rectilinear coordinates.

e. [2 points] if $f(x, y) = xy$ and $C: p(t) = (t, 1 + t^2)$ for $0 \leq t \leq 2$ then $\int_C \nabla f dL = 10$.

8. [6 points] Let $f(x, y)$ be a function of two variables.
- [3 points] State the limit definition of $\frac{df}{dx}(0, 1)$.

b. [3 points] Let $n = (1, 2)$. State the limit definition of $\frac{df}{dn}(0, 1)$.

9. [10 points] Let $f(x, y) = (\sqrt{xy}, \sqrt{x+y})$.

a. [5 points] What is the equation for the tangent plane at $(2, 2)$?

b. [5 points] Use part (a) to approximate $f(1, 2)$.

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10. [7 points] Find the critical points and classify them (local min, local max, saddle point) of the function $f(x, y) = x + y - \ln(xy)$.

11. [2 points] What was your favorite topic this semester?