1. Consider the function $f(x) = x^3 \ln x$ on the interval $[1/e, e^2]$.

   (a) Find the $x$- and $y$-coordinates of any and all local extrema and classify each as a local maximum or local minimum.

   (b) Find the $x$- and $y$-coordinates of any and all global extrema and classify each as a global maximum or global minimum.

   (c) Find the $x$-coordinate(s) of any and all inflection points.

2. You are watching a plane flying toward your position at a constant height of 3 miles and a speed of 500 miles per hour relative to the ground. At the moment when the plane is 5 miles from you (diagonally), at what rate is the angle of your vision toward the plane changing?
3. Your company is mass-producing a cylindrical container. The flat portion (top and bottom) costs 3 cents per square inch and the curved (lateral) portion costs 5 cents per square inch. If your budget is $9.00 per container, what dimensions will give the largest volume?

area of circle = \( \pi r^2 \)  
lateral area of cylinder = \( 2\pi rh \)  
volume of cylinder = \( \pi r^2 h \)

4. State the Intermediate Value Theorem and use it to show that \( f(x) = x^3 - x + 1 \) has a root in \([-2, 0]\).

5. (Sections A and B may omit this problem.) Use Newton’s Method to find a root of \( f(x) = x^3 - x + 1 \) correct to three decimal places.
6. What (if anything) does the Extreme Value Theorem say about $f(x) = x^2$ on each of the following intervals?

(a) $[1, 4]$

(b) $(1, 4)$

7. State the Mean Value Theorem and find the value of the constant $c$ that the theorem specifies for $f(x) = x^3 + x$ on $[0, 3]$.

8. Find the following.

(a) $\int_{1}^{7} \frac{3}{x} \, dx$

(b) $\int_{1}^{4} (1 + 2x + x^3 + 4\sqrt{x} + \frac{1}{x^3}) \, dx$

(c) $\int_{0}^{2} e^{3x} \, dx$

(d) $\int_{-2}^{2} \sqrt{4 - x^2} \, dx$

(e) $\frac{d}{dx} \int_{1}^{x} \sin \sqrt{t} \, dt$

(f) $\lim_{n \to \infty} \frac{2}{n} \sum_{k=1}^{n} \left(1 + \frac{2k}{n}\right)^2$

9. Water is leaking out of a tank at a decreasing rate. Find an overestimate and underestimate for the total amount that leaked out during these 8 minutes.

<table>
<thead>
<tr>
<th>time (min)</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>rate (gal/min)</td>
<td>15</td>
<td>11</td>
<td>8</td>
<td>4</td>
<td>3</td>
</tr>
</tbody>
</table>
10. Consider the graph of $f(t)$ shown. It is made of straight lines and a semicircle.

Let $G(x) = \int_0^x f(t) \, dt$ and $H(x) = \int_{-3}^x f(t) \, dt$.

(a) Compute $G(2)$, $G(4)$, and $H(4)$.

(b) Where is $G$ increasing? Where is $G$ decreasing?

(c) Where is $G$ concave up? Where is $G$ concave down?

(d) At what $x$-value(s) does $G$ have a local maximum? At what $x$-value(s) does $G$ have a local minimum?

(e) Find a formula that relates $G$ and $H$.

(f) How would your answers to (b), (c), and (d) change if the questions were about $H$ instead of $G$?

11. Use sigma notation to express $L_{10}$ and $M_{10}$ as approximations to $\int_{20}^{60} \ln x \, dx$. 