Math 105: Review for Final Exam, Part II

1. Consider the function $f(x) = x^3 \ln x$ on the interval $[1/e, e^2]$.

   (a) Find the x- and y-coordinates of any and all local extrema and classify each as a local maximum or local minimum.

   (b) Find the x- and y-coordinates of any and all global extrema and classify each as a global maximum or global minimum.

   (c) Find the x-coordinate(s) of any and all inflection points.

2. You are watching a plane flying toward your position at a constant height of 3 miles and a speed of 500 miles per hour relative to the ground. At the moment when the plane is 5 miles from you (diagonally), at what rate is the angle of your vision toward the plane changing?

3. Your company is mass-producing a cylindrical container. The flat portion (top and bottom) costs 3 cents per square inch and the curved (lateral) portion costs 5 cents per square inch. If your budget is \$9.00 per container, what dimensions will give the largest volume?

   area of circle $= \pi r^2$ lateral area of cylinder $= 2\pi rh$ volume of cylinder $= \pi r^2 h$

4. State the Intermediate Value Theorem and use it to show that $f(x) = x^3 - x + 1$ has a root in $[-2, 0]$.

5. (Sections A and B may omit this problem.) Use Newton's Method to find a root of $f(x) = x^3 - x + 1$ correct to three decimal places.

6. What (if anything) does the Extreme Value Theorem say about $f(x) = x^2$ on each of the following intervals?

   (a) $[1, 4]$

   (b) $(1, 4)$

7. State the Mean Value Theorem and find the value of the constant c that the theorem specifies for $f(x) = x^3 + x$ on $[0, 3]$.

8. Find the following.

   (a) $\displaystyle\int_1^7 \frac{3}{x}\,dx$

   (b) $\displaystyle\int_1^4 (1 + 2x + x^3 + 4\sqrt{x} + \frac{1}{x^5})\,dx$

   (c) $\displaystyle\int_0^2 e^{3x}\,dx$

   (d) $\displaystyle\int_{-2}^2 \sqrt{4 - x^2}\,dx$

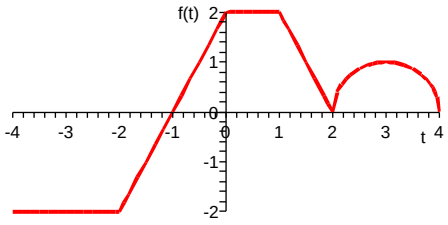
   (e) $\displaystyle\frac{d}{dx}\int_1^x \sin\sqrt{t}\,dt$

   (f) $\displaystyle\lim_{n\to\infty} \frac{2}{n}\sum_{k=1}^{n}\left(1 + \frac{2k}{n}\right)^2$

9. Water is leaking out of a tank at a decreasing rate. Find an overestimate and underestimate for the total amount that leaked out during these 8 minutes.

time (min)	0	2	4	6	8
rate (gal/min)	15	11	8	4	3

10. Consider the graph of $f(t)$ shown. It is made of straight lines and a semicircle.



Let $G(x) = \int_0^x f(t)\ dt$ and $H(x) = \int_{-3}^x f(t)\ dt$.

(a) Compute $G(2)$, $G(4)$, and $H(4)$.

(b) Where is G increasing? Where is G decreasing?

(c) Where is G concave up? Where is G concave down?

(d) At what x-value(s) does G have a local maximum? At what x-value(s) does G have a local minimum?

(e) Find a formula that relates G and H.

(f) How would your answers to (b), (c), and (d) change if the questions were about H instead of G?

11. Use sigma notation to express L_{10} and M_{10} as approximations to $\int_{20}^{60} \ln x\ dx$.