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I ___ II ___ III ___ IV ___ V ___ VI ___ VII ___ VIII ___ IX ___ X ___ XI ___

XII ___ XIII ___ XIV ___ TOTAL _____

December 10
2008

Mathematics 106a
Calculus II
Final Examination

Mr. Balcomb

(5) I. Write the fraction $\frac{1}{x^2(x-1)}$ as the sum of rational functions.

(5) II. If $\int_1^{\infty} \frac{1}{x^p} dx$ converges to $1/2$, what is the value of p ? Justify your answer.

(15) III. Find the integrals:

A. $\int 12x^2 \cos(x^3) dx$.

B. $\int \frac{3x^2 + 1}{x(x^2 + 1)} dx$.

C. $\int_0^{\pi/4} \tan^3 x \sec^2 x dx$.

(10) IV. For $f(x) = \sqrt[3]{x+1}$

A. Give the first degree Taylor polynomial for f at 7.

B. Use this polynomial to estimate $\sqrt[3]{9}$.

C. What is the largest possible error that could have occurred in your estimate in part B?

Recall that if you use the Taylor polynomial of degree n at x_0 to approximate

$f(x)$ for x in an interval I containing x_0 then $\frac{K_{n+1} |x - x_0|^{n+1}}{(n+1)!}$ is an upper

bound for the approximation error. [K_{n+1} is an upper bound for the absolute value of the $(n+1)$ st derivative of f on I .]

(5) V. Starting with the Maclaurin series for $\frac{1}{1+x}$,

A. find a power series expression for $\int \frac{1}{1+x^4} dx$.

B. For what values of x does this formula hold?

(10) VI. A spherical tank of radius 20 feet is buried 8 feet below ground and is filled to a height of 13 feet from the bottom of the tank with water (62.5 pounds per cubic foot). Write an integral equal to the work done in pumping all the water to ground level.

(5) VII. Consider the region bounded by $y = 0$, $x = 2$, and $y = x^2$. Write an integral equal to the volume of the object created when the region is revolved about the line $x = 7$.

(10) VIII. Find the solution that passes through $(0, 2)$ for the equations:

A. $\frac{dy}{dx} = \frac{\cos x}{y}$

B. $\frac{dy}{dx} = xe^x$

(5) IX. Let $I = \int_0^2 x^3 dx$.

A. Use the Fundamental Theorem of Calculus to evaluate I exactly.

B. Compute the approximating sum R_4

C. Compute the approximation error $|I - R_4|$

(5) X. Write (but do not evaluate) an integral that gives the arc length of the graph of $f(x) = \sin x$ over the interval $[0, \pi/2]$.

(10) XI. Do these integrals converge? Justify your answers.

A. $\int_2^{\infty} \frac{3}{1+x^3} dx$

B. $\int_1^{\infty} \frac{\cos x}{x^{3/2}} dx$

(5) XII. If the series $\sum_{k=2}^{\infty} \left(\frac{3}{4}\right)^k$ converges, to what does it converge? If it diverges, explain why.

(5) XIII. Test to determine whether the series $\sum_{n=1}^{\infty} \frac{2n}{12n+5}$ converges or diverges and explain why.

(5) XIV. For the series $\sum_{n=1}^{\infty} (-1)^{n+1} \left(\frac{x-1}{2} \right)^n :$

A. Give its radius of convergence:

B. Give its interval of convergence:

C. Give a value of x for which it diverges: