# Math 106: Review for Final Exam, Part I

1. Find the following. [See Review for Exam II for integration tips and strategies.]

   (a) $\displaystyle\int 12x^2\cos(x^3)\,dx$

   (b) $\displaystyle\int_0^\infty xe^{-3x}\,dx$

   (c) $\displaystyle\int_0^6 \frac{dx}{(x-4)^2}$

   (d) $\displaystyle\int \frac{3x^2+2x-5}{(x^2+1)(x-4)}\,dx$

   (e) $\displaystyle\int_0^{\pi/3} \tan^3 x\sec^5 x\,dx$

(f) $\int \sqrt{25-x^2}\,dx$

2. Find the best possible left, right, midpoint, trapezoidal, and Simpson's approximations to $\int_{-2}^{0} f(x)\,dx$ given the data in the table below. [8:00 and 9:30 sections may omit Simpson's approximation.]

| $x$ | -2 | -1.5 | -1 | -0.5 | 0 |
|---|---|---|---|---|---|
| $f(x)$ | 2 | 3 | 6 | 10 | 11 |

3. If you use numerical integration to estimate $\int_a^b \ln x\,dx$, how would the following be ordered from least to greatest? $L_{100}$, $R_{100}$, $M_{100}$, $T_{100}$, $S_{200}$. [8:00 and 9:30 sections may omit $S_{200}$.]

What can you say with certainty about where $\int_a^b \ln x\,dx$ would fit into your ordering?

4. Find bounds for each of the following errors if $I = \int_0^2 e^{-3x}\,dx$.

   (a) $|I - L_{100}|$

   (b) $|I - T_{100}|$

   (c) $|I - M_{100}|$

5. If $I = \int_0^2 e^{-3x}\,dx$, how many subdivisions are required to obtain a midpoint sum approximation with error of at most 1/1,000,000?

6. Use Euler's Method with 3 steps to estimate $y(3/4)$ if $dy/dx = y - 3$ and $y(0) = 1$.

7. Write an integral equal to the area between $y = 2x + 3$ and $y = x^2 + 7x - 3$.

8. Compute the arc length of $y = \sqrt{1 - x^2}$ from $x = 0$ to $x = 1/2$.

9. Consider the region bounded by $y = 0$, $x = 2$, and $y = x^2$. Write an integral equal to the volume of the object created when the region is revolved about

(a) the $x$-axis

(b) the line $x = 5$

10. A spherical tank of radius 8 feet is buried 5 feet below ground and filled to a height of 11 feet with gasoline (42 pounds per cubic foot). Write an integral equal to the work done in pumping all the gasoline to ground level.

11. Find the solution to $\dfrac{dy}{dx} = \dfrac{\cos x}{y^2}$ that passes through $(0, 2)$.