

Math 105: Review for Final Exam, Part I

1. Consider the function $f(x) = \frac{3}{5 - 2x}$.

(a) Is this function continuous on the interval $(-\infty, \infty)$? Explain.

(b) Compute the average rate of change of f on $[2, 2.01]$.

(c) Using the limit definition of the derivative, compute $f'(x)$.

(d) Find the equation of the tangent line to f at $x = 2$.

2. Given that $f(0) = 2$, $g(0) = 3$, $f'(0) = 5$, $g'(0) = 7$, and $f'(3) = \pi$ compute the following.

(a) $h'(0)$ if $h(x) = f(x)g(x)$

(b) $j'(0)$ if $j(x) = \frac{f(x)}{g(x)}$

(c) $k'(0)$ if $k(x) = f(g(x))$

3. Compute dy/dx for each of the following.

(a) $y = x^5 + 5^x + e^5 + \frac{x}{5} + \frac{5}{x} + \frac{5}{\sqrt[5]{x}} + \ln(5x) + \arctan(5x) + \ln(5) + \sin 5$

(b) $y = \sqrt[3]{x} \cos(7x^3)$

(c) $y = \frac{e^x + e^\pi}{\tan 4 - 7x}$

(d) $y = \tan(e^{x^2} \arcsin(5x))$

(e) $y^3 + yx^2 + x^2 = 3y^2$

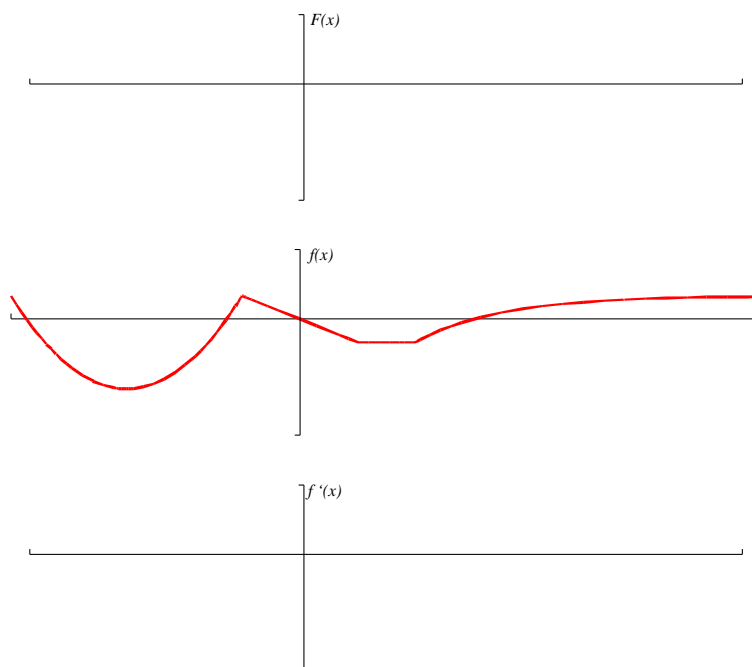
4. The rate of change of the temperature $T(t)$ (in degrees Fahrenheit) of a piece of wedding cake t hours after it is placed in a very cold freezer is proportional to the the temperature of the cake. The cake's temperature when placed in the freezer is 70 degrees. One hour later, its temperature is 60 degrees.

(a) Write a differential equation whose solution is $T(t)$. Your equation may have an unknown constant in it.

(b) Solve your differential equation.

(c) When will the temperature of the cake reach 0 degrees?

5. Given the graph of f , sketch a graph of f' and a graph of F , an antiderivative of f such that $F(0) = -1$.



6. Shown below is a graph of f' on its entire domain. The graph is NOT f .

At which x -value(s)

- (a) does f have a stationary point?
- (b) does f have a local max?
- (c) does f have a local min?
- (d) does f' have a stationary point?
- (e) does f' have a local max?
- (f) does f' have a local min?
- (g) is f greatest?
- (h) is f least?
- (i) is f' greatest?
- (j) is f' least?
- (k) is f'' greatest?
- (l) is f'' least?

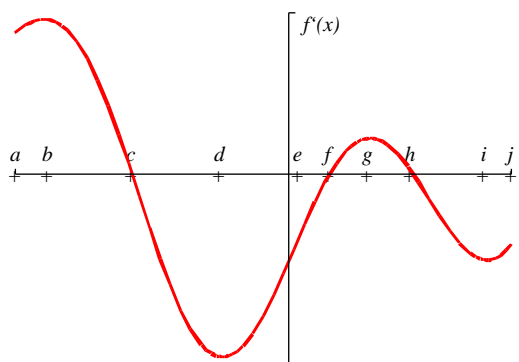
(b) f decreasing?

(c) f' increasing?

(d) f' decreasing?

(e) f concave up?

(f) f concave down?



On what interval(s) is

- (a) f increasing?

7. Is $y = 7e^{3x}$ a solution to the differential equation $y'' + 2y' - 15y = 0$? Explain.

8. Rewrite $\sin(\arctan(5x))$ as an algebraic expression.

9. Evaluate the following limits.

(a) $\lim_{x \rightarrow \infty} \frac{x^2}{\ln x}$

(b) $\lim_{x \rightarrow 0} \frac{\sin(12x) - 12x}{x^3}$

(c) $\lim_{x \rightarrow 0} \frac{e^x - 1}{\cos x}$

(d) $\lim_{x \rightarrow 5} \frac{35 - 7x}{2x - 10}$