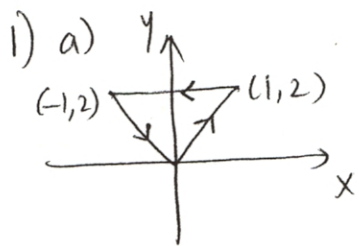


MATH 206 Section A

Solutions to Review for Chapters 5 and 6



$$\frac{\partial F_2}{\partial x} = \frac{\partial (3y^2)}{\partial x} = 0 \quad \frac{\partial F_1}{\partial y} = \frac{\partial (2x+y)}{\partial y} = 1$$

$$\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} \neq 0. \text{ So } \vec{F} \text{ is not path-independent}$$

C : closed curve in \mathbb{R}^2 , domain of \vec{F} is simply connected.

Can use Green's theorem.

$$\int_C \vec{F} \cdot d\vec{x} = \iint_R \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} \right) dA = \iint_R (-1) dA = (-1) (\text{area of triangle}) = -2.$$

(R : triangular region bounded by triangle)

1) (b) $\text{curl } \vec{F} = -10y\vec{i} - 5x\vec{k} \neq \vec{0}$

\vec{F} is not path-independent.

Parametrize C : $f(t) = (t^2, t, -2)$, $0 \leq t \leq 1$.

$$\vec{F}(f(t)) = (5t^3, -20t, -2) \quad f'(t) = (2t, 1, 0)$$

$$\int_C \vec{F} \cdot d\vec{x} = \int_0^1 (5t^3, -20t, -2) \cdot (2t, 1, 0) dt = \int_0^1 (10t^4 - 20t) dt = -8$$

1) (c) $\text{curl } \vec{F} = \vec{0}$. \vec{F} is path-independent.

Find potential function $f(x, y, z)$ such that $\text{grad } f = \vec{F}$.

$$\frac{\partial f}{\partial x} = \cos x \text{ gives } f(x, y, z) = \sin x + g(y, z). \text{ So } \frac{\partial f}{\partial y} = \frac{\partial g}{\partial y}.$$

$$\text{But } \frac{\partial f}{\partial y} \text{ has to be } e^y. \text{ So } \frac{\partial g}{\partial y} = e^y \text{ so } g(y, z) = e^y + h(z).$$

$$\text{So } f(x, y, z) = \sin x + e^y + h(z) \quad \frac{\partial f}{\partial z} = \frac{\partial h}{\partial z} \text{ which has to be } 2z$$

So $h(z) = z^2 + \text{constant}$. Choose constant to be 0.

$f(x, y, z) = \sin x + e^y + z^2$. (Check $\text{grad } f = \vec{F}$. You can also find f by guess-and-check.)