

Math 106: Review for Final Exam, Part II

1. Use a second-degree Taylor polynomial to estimate $\sqrt[3]{28}$.
2. What is the largest possible error that could have occurred in your previous estimate?
3. Use a comparison to show whether each of the following converges or diverges. If an integral converges, give a good upper bound for its value.

(a) $\int_1^{\infty} \frac{7 + 5 \sin x}{x^2} dx$

(b) $\int_1^{\infty} \frac{1 + 3x^2 + 2x^3}{\sqrt[3]{10x^{12} + 17x^{10}}} dx$

4. Decide if each of the following sequences $\{a_k\}_{k=1}^{\infty}$ converges or diverges. If a sequence converges, compute its limit.

(a) $a_k = 3 + \frac{1}{10^k}$

(b) $a_k = (-1)^k$

(c) $a_k = \frac{3 + 5k}{7 + 2k}$

Strategy. The following is a good order in which to consider the various series convergence tests.

- Do the individual terms approach 0? If they don't approach 0, the n th Term Test tells you the series must diverge. If they do approach 0, try another test.
 - Is the series geometric? (That is, do you multiply by the same constant r to get from each term to the next?) If so, the series converges if $|r| < 1$ and diverges otherwise.
 - Does the series contain something such as $(-1)^k$ or $(-1)^{k+1}$ or $\cos(k\pi)$ that makes its terms alternate? If so, try the Alternating Series Test.
 - Does the series contain a factorial ($k!$) or exponential (such as 2^k or e^k)? If so, try the Ratio Test.
 - If the series has positive terms, does it remind you of a simpler series (especially a p -series: powers of k such as $1/k$ or $1/k^2$)? If so, try the Comparison Test.
 - Is the formula something you can integrate easily? If so, try the Integral Test.
5. Decide if each of the following series converges or diverges. If a series converges, find its value.
- $3.1 + 3.01 + 3.001 + 3.0001 + \dots$

(b) $1 + 1/2 + 1/3 + 1/4 + \dots$

(c) $5 - 5/3 + 5/9 - 5/27 + \dots$

6. Decide if each of the following series converges or diverges. If a series converges, find upper and lower bounds for its value.

(a)
$$\sum_{k=1}^{\infty} \frac{(-1)^k}{\sqrt[3]{k+1}}$$

(b)
$$\sum_{k=1}^{\infty} \frac{(2k)!}{3^k (k!)^2}$$

(c)
$$\sum_{k=1}^{\infty} \left(\frac{1}{100} + \frac{1}{k^5} \right)$$

(d)
$$\sum_{k=1}^{\infty} \frac{\sqrt{9k^8 + 5k^6}}{12k^5 + 3}$$

(e)
$$\sum_{k=2}^{\infty} \frac{1}{k(\ln(k))^2}$$

7. Does the first series from the previous problem converge absolutely or conditionally?

8. Compute the radius and interval (including endpoints) of convergence for $\sum_{k=1}^{\infty} \frac{(x+3)^k}{k \cdot 5^k}$.

9. Find the complete Taylor series (in summation notation) for $f(x) = \ln(1 - x)$ about $x = 0$.

10. Evaluate the following exactly.

(a) $1 - 1 + \frac{1}{2} - \frac{1}{6} + \frac{1}{24} - \frac{1}{120} + \dots$

(b) $\frac{8}{3} - \frac{8}{9} + \frac{8}{27} - \frac{8}{81} + \dots$

(c) $1 - \frac{\pi^2}{2} + \frac{\pi^4}{24} - \frac{\pi^6}{720} + \dots$

11. (a) Write the complete series equal to $\int_0^1 e^{-x^2} dx$ and show that it converges.

(b) If $f(x) = e^{-x^2}$, what is $f^{(400)}(0)$? What is $f^{(401)}(0)$?