NAME____

I____II____III____IV____V____VI____VII____VIII____IX___ X____TOTAL ___________

December 9 2005 | Mathematics 206a
Multivariable Calculus
Final Examination | Mr. Haines

(10) I. Give a parametrization of

A. the line segment that connects the points (1, 0, 3) and (4, 5, 5) in $\Re^3$.

B. the surface bounded by the triangle in $\Re^3$ located in the first octant and cut off by the plane whose equation is $x + y + z = 1$.

(5) II. Compute the area of the triangle in $\Re^3$ which has vertices (1, 1, 1), (4, 5, 6), and (3, 7, 5).

(25) III. Let M be the surface in $\Re^3$ parametrized by $\mathbf{f}(s,t) = (s, s\cos t, s\sin t)$ for $0 \le s \le 2$ and $0 \le t \le \pi$.

A. Give a coordinate equation for M in terms of *x*, *y*, and *z*. Describe M in words.

B. Calculate a unit normal to M at the point (1, 0, 1).

C. Give an equation of the tangent plane to M at the point (1, 0, 1).

(This continues the problem from the previous page ….)

D. Calculate the surface area of M .

E. If $\mathbf{F}(x, y, z) = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$, calculate the value of the surface integral $\iint_M \mathbf{F} \bullet \mathbf{n} d\sigma$.

(15) IV. The curve C is parametrized by $\mathbf{f}(t) = (t^2 - 1,\ t^2 + 1, t^2)$ starting at $t = 0$ and ending at $t = 1$.

A. Compute the length of C.

B. Compute the line integral of $\mathbf{F}(x, y, z) = y^4\mathbf{i} + 4xy^3\mathbf{j} + 2z\mathbf{k}$ over C.

C. Compute the line integral of $\mathbf{F}(x, y, z) = y^4\mathbf{i} + 4xy^3\mathbf{j} + 2z\mathbf{k}$ over the curve that is the ellipse obtained by intersecting the cylinder $x^2 + y^2 = 1$ with the plane $3x + 5y - 2z = 0$.

(5) V. Compute $\int_0^2 \int_0^{\sqrt{4-x^2}} \int_0^{\sqrt{4-x^2-y^2}} x^2 + y^2 + z^2 dzdydx$ by converting to spherical coordinates.

(5) VI. $\mathbf{f}(t) = (\cos t, \sin t, t)$ with $0 \le t$ is a parametrization of a helix in 3-space. Give the equation of the tangent line to this path at the point where $t = \frac{\pi}{2}$.

(5) VII. Suppose $\mathbf{f}(x, y, z) = (xy^2z, xz(y+1))$ and $\mathbf{a}$ = (1, 1, 1)

A) Calculate the Jacobian matrix of $\mathbf{f}$ at $\mathbf{a}$.

B) Calculate the total derivative of $\mathbf{f}$ at $\mathbf{a}$.

(10) VIII. For the quadratic form $p(x, y) = x^2 + 3xy + y^2$

A. Give a symmetric matrix S that is a matrix of this quadratic form.

B. Calculate the Hessian for p at (0, 0).

C. Give the second degree Taylor polynomial for p at (0, 0).

(10) IX. Let $\mathbf{F}(x, y) = (y^2, y + 2x)$. Let R be the triangular region in the first quadrant bounded by the curves $y = 0$, $x = 1$, and $y = x$. Use Green's Theorem to calculate $\oint_{\partial R} \mathbf{F} \bullet d\mathbf{x}$.

(10) X. Use the Divergence theorem to calculate $\oiint_{\partial S} \mathbf{F} \bullet \mathbf{n}\, d\sigma$, where $\mathbf{F} = 2x\mathbf{i} + 3y\mathbf{j} + 5z\mathbf{k}$ and S is the unit cube in the first octant. ($S = [0,1]\times[0,1]\times[0,1]$).