

NAME: \_\_\_\_\_

YOUR GRADE IS BASED ON THE PROCESS AS WELL AS THE FINAL RESULT. SHOW ALL YOUR STEPS CLEARLY SO YOU WILL BE ELIGIBLE FOR THE MOST PARTIAL CREDIT. YOU MAY USE A CALCULATOR, BUT NO NOTES, BOOKS, OR OTHER STUDENTS. GOOD LUCK!

1.) (10 pts.)

a.) (5 pts.) What does the Extreme Value Theorem say about the function  $f(x) = \sqrt{9 - x^2}$  on the interval  $[0, 3]$ ?

b.) (5 pts.) Use either geometry or the Fundamental Theorem of Calculus to compute  $\int_0^3 \sqrt{9 - x^2} dx$ .

2.) (15 pts.) A cylindrical can (with top) is to be made to hold 1 liter of oil. Find the dimensions that will minimize the cost of the metal to manufacture the can.

3.) (15 pts.) Given the integral  $\int_0^\pi \sin x \, dx$ ,

a.) (4 pts.) compute the exact integral using the Fundamental Theorem of Calculus;

b.) (4 pts.) approximate the integral using a trapezoidal sum with 2 subintervals (include a picture);

c.) (4 pts.) approximate the integral using a left-endpoint sum with 4 subintervals (include a picture);

d.) (3 pts.) write your answer to part (c.) in  $\Sigma$ -notation.

4.) (15 pts.)

a.) (5 pts.) State the hypotheses of the Mean Value Theorem.

b.) (5 pts.) State the conclusion of the Mean Value Theorem.

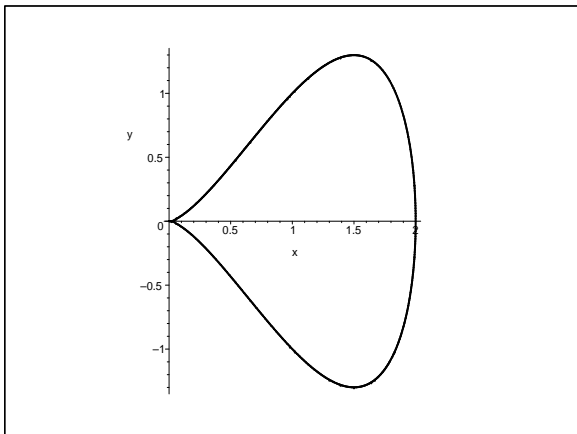
c.) (5 pts.) Sketch a function and appropriate interval that satisfy the Mean Value Theorem. Be sure to show that both sides of the equation in the MVT's conclusion are equal in your sketch.

5.) (15 pts.) The equation  $y^2 = x^3(2 - x)$  has the shape of a *piriform*, shown in the graph at the bottom of this page.

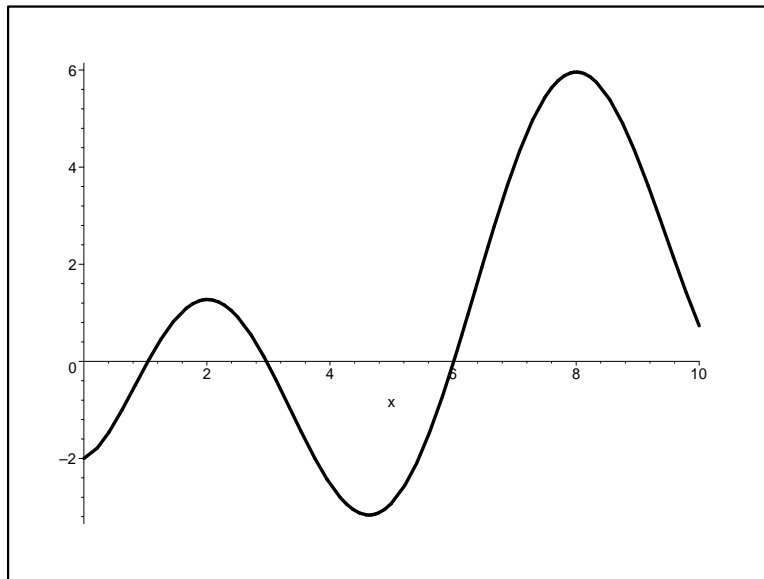
a.) (5 pts.) Compute  $\frac{dy}{dx}$  for the piriform equation.

b.) (5 pts.) Find the equation of the tangent line to the piriform at the point  $(1, 1)$ .

c.) (5 pts.) Sketch the tangent line at  $(1, 1)$  on the graph below. Does it agree with your result in part (b.)? Explain.



6.) (15 pts.) The graph below is  $f(x)$  and  $F(x) = \int_0^x f(t) dt$ . Use the graph to answer the following questions.



a.) (4 pts.) On what intervals is  $F$  concave up?

b.) (4 pts.) At which  $x$ -values does  $F$  have a local minimum?

c.) (4 pts.) On what intervals is  $F$  decreasing?

d.) (3 pts.) On what intervals is  $F'$  decreasing?

7.) (15 pts.) Draw a single function  $f(x)$  in which all of the following are true:

i.)  $\lim_{x \rightarrow -3^-} f(x) = -1$

ii.)  $\lim_{x \rightarrow -3^+} f(x) = 2$

iii.)  $\lim_{x \rightarrow 1} f(x) = \infty$

iv.)  $\lim_{x \rightarrow \infty} f(x) = -5$

**BONUS: (5 pts.)** Which high school dropout was able to teach us something about Calculus AND what did we learn from him?