1. Consider the function $f(x) = x^6 - 2x^3$ on the interval $[-2, 2]$.

(a) Find the $x$- and $y$-coordinates of any and all local extrema and classify each as a local maximum or local minimum.

(b) Find the $x$- and $y$-coordinates of any and all global extrema and classify each as a global maximum or global minimum.

(c) Find the $x$-coordinate(s) of any and all inflection points.

2. How would your answers to the previous question have changed if the domain of $f$ were all reals?
3. Your company is mass-producing a cylindrical container. The flat portion (top and bottom) costs 3 cents per square inch and the curved (lateral) portion costs 5 cents per square inch. If your budget is $9.00 per container, what dimensions will give the largest volume?

area of circle = $\pi r^2$   lateral area of cylinder = $2\pi rh$   volume of cylinder = $\pi r^2 h$

4. Use Newton’s Method to find a root of $f(x) = x^3 - x + 1$ correct to three decimal places.

5. Find the Taylor polynomial of order three for $f(x) = \sqrt[3]{x}$ based at $x = 8$. 
6. State the Intermediate Value Theorem and use it to show that \( f(x) = 4x^7 + 3x^3 - 5 \) has a root in \([0, 1]\).

7. What (if anything) does the Extreme Value Theorem say about \( f(x) = x^2 \) on each of the following intervals?
   (a) \([1, 4]\)
   (b) \((1, 4)\)

8. State the Mean Value Theorem and find the value of the constant \( c \) that the theorem specifies for \( f(x) = x^3 + x \) on \([0, 3]\).

9. Find the following.
   (a) \( \int_1^7 \frac{3}{x} \, dx \)
   (b) \( \int_0^2 e^{3x} \, dx \)
   (c) \( \int_{-2}^2 \sqrt{4 - x^2} \, dx \)
   (d) \( \frac{d}{dx} \int_1^x \sin \sqrt{t} \, dt \)

10. Water is leaking out of a tank at a decreasing rate. Find an overestimate and underestimate for the total amount that leaked out during these 8 minutes.

<table>
<thead>
<tr>
<th>time (min)</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>rate (gal/min)</td>
<td>15</td>
<td>11</td>
<td>8</td>
<td>4</td>
<td>3</td>
</tr>
</tbody>
</table>
11. The rate of change of a room’s temperature is \( r(t) = t^2 - 9 \) degrees per hour on the interval \([0, 4]\) hours. At \( t = 0 \), the temperature is 70 degrees. (Remember that \( r \) is the derivative of the temperature function.)

(a) When on this interval is the temperature rising? falling?

(b) What is the maximum temperature on this interval and when does it occur?

(c) What is the minimum temperature on this interval and when does it occur?

(d) What is the average rate of change of the temperature on this interval?

12. An object is launched vertically into the air from ground level with an initial velocity of 160 feet per second. Gravity causes a downward acceleration of 32 ft/sec/sec. What is its velocity when it first reaches a height of 256 feet? When it next reaches this same height?