1. Consider the function \( f(x) = \frac{3}{5 - 2x} \).

(a) Is this function continuous on the domain \((-\infty, \infty)\)? Explain.

No. It is discontinuous at \( x = 5/2 \), where \( f \) is undefined.

(b) Compute the average rate of change of \( f \) on \([1.5, 2]\).

\[
\frac{f(2) - f(1.5)}{2 - 1.5} = \frac{3 - 1.5}{0.5} = 3
\]

(c) Using the limit definition of the derivative, compute \( f'(x) \).

\[
f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\frac{3}{5 - 2(x+h)} - \frac{3}{5 - 2x}}{h} = \lim_{h \to 0} \frac{6}{(5 - 2x)(5 - 2x - 2h)} = \frac{6}{(5 - 2x)^2}
\]

(d) Find the equation of the tangent line to \( f \) at \( x = 2 \).

Want \( y = mx + b \).

\[
m = f'(2) = \frac{6}{(5 - 2)^2} = 6
\]

Need a point.

\[
x = 2 \Rightarrow y = f(2) = \frac{3}{5 - 2} = 3
\]

Thus \( 3 = 6\cdot2 + b \)

\[
-9 = b \Rightarrow y = 6x - 9
\]

2. Given that \( f(0) = 2, g(0) = 3, f'(0) = 5, g'(0) = 7, \) and \( f'(3) = \pi \) compute the following.

(a) \( h'(0) \) if \( h(x) = f(x)g(x) \)

\[
h'(0) = f'(0)g(0) + f(0)g'(0) = 5 \cdot 3 + 2 \cdot 7 = 29
\]

(b) \( j'(0) \) if \( j(x) = \frac{f(x)}{g(x)} \)

\[
j'(0) = \frac{f'(0)g(0) - f(0)g'(0)}{[g(0)]^2} = \frac{5 \cdot 3 - 2 \cdot 7}{3^2} = \frac{1}{9}
\]

(c) \( k'(0) \) if \( k(x) = f(g(x)) \)

\[
k'(0) = f'(g(0)) \cdot g'(0) = f'(3) \cdot 7 = \pi \cdot 7 = \frac{7\pi}{2}
\]

3. Sketch a graph of a function which is always positive and decreasing and which satisfies the following:

\[
\lim_{x \to -\infty} f(x) = \infty; \lim_{x \to -1} f(x) = 2; \lim_{x \to 1} f(x) = 5; \lim_{x \to 1^+} f(x) = 4
\]

There are other possible correct answers.
4. Compute $dy/dx$ for each of the following.

(a) $y = x^2 + 2x + e^x + \frac{x}{2} + 2 + \ln(2x) + \arctan(2x) + \ln(2) + \sin 2$

\[
\frac{dy}{dx} = 2x + \ln 2 \cdot 2^x + 0 + \frac{1}{2} + 2(-1)x^{-2} + \frac{1}{2} \cdot 2 + \frac{1}{1 + (2x)^2} \cdot 2 + 0 + 0
\]

(b) $y = \sqrt{x} \cos(7x^3)$

Product Rule

\[
\frac{dy}{dx} = 5x \cdot (-\sin(7x^3)) \cdot 21x^2 + \frac{1}{2} \cdot x^{-11/2} \cos(7x^3) = -21x^{5/2} \sin(7x^3) + \frac{\cos(7x^3)}{21x}
\]

(c) $y = \frac{e^x + \pi}{\tan 4 - 7x}$

Quotient Rule

\[
\frac{dy}{dx} = \frac{e^x \tan 4 - 7x - (e^x + \pi)(-7)}{(\tan 4 - 7x)^2}
\]

(d) $y = \tan(e^{x^2} \arcsin(5x))$

\[
\frac{dy}{dx} = \sec^2(e^{x^2} \arcsin(5x)) \cdot e^{x^2} \arcsin(5x) \cdot (x^2 \cdot \frac{1}{\sqrt{1 - 25x^2}} + 2x \arcsin(5x))
\]

(c) $y^3 + y^2 + x^2 = 3y^2$

Implicit:

\[
3y^2 \frac{dy}{dx} + \frac{dy}{dx} x^2 + y \cdot 2x + 2x = 6y \frac{dy}{dx}
\]

\[
\frac{dy}{dx} \left(3y^2 + x^2 - 6y\right) = -2x - 2xy \quad \Rightarrow \quad \frac{dy}{dx} = \frac{-2x - 2xy}{3y^2 + x^2 - 6y}
\]

5. Consider the differential equation $y' = -3y$.

(a) Sketch the slope field for this DE.

(b) Verify that $y = Ce^{-3x}$ is a solution for all values of $C$.

If $y = Ce^{-3x}$, then $y' = Ce^{-3x} \cdot (-3)$ or $-3Ce^{-3x}$.

And $-3y$ is also $-3Ce^{-3x}$, so both sides of the DE are equal.

(c) Find the solution that passes through $(1, 5)$.

$5 = Ce^{-3} \Rightarrow C = 5e^3$

\[
\Rightarrow y = 5e^{-3x}
\]
6. Given the graph of \( f \), sketch a graph of \( f' \) and a graph of \( F \), an antiderivative of \( f \) such that \( F(0) = -1 \).

7. The graph shown is \( f' \), NOT \( f \). Answer the questions below.

At which labeled point(s) does

\( C_i \)  
(a) \( f \) have a stationary point?
\( C \)  
(b) \( f \) have a local max?
\( H \)  
(c) \( f \) have a local min?
\( D_i \)  
(d) \( f'' \) have a stationary point?
\( B_i \)  
(e) \( f' \) have a local max?
\( F \)  
(f) \( f' \) have a local min?
\( C \)  
(g) \( f \) have a global max?
\( H \)  
(h) \( f \) have a global min?
\( I \)  
(i) \( f' \) have a global max?
\( F \)  
(j) \( f'' \) have a global min?
\( G \)  
(k) \( f'' \) have a global max?
\( D \)  
(l) \( f'' \) have a global min?

On what interval(s) is

\[ [A,C) \cup (H,J] \]  
(a) \( f \) increasing?
\[ (C,H) \]  
(b) \( f \) decreasing?
\[ (A,B) \cup (F,I) \]  
(c) \( f' \) increasing?
\[ (B,F) \cup (I,J) \]  
(d) \( f' \) decreasing
\[ [A,B) \cup (F,I) \]  
(e) \( f \) concave up?
\[ (B,F) \cup (I,J) \]  
(f) \( f \) concave down?
\[ (D,G) \]  
(g) \( f' \) concave up?
\[ [A,D) \cup (G,J] \]  
(h) \( f' \) concave down?

On the same set of axes, sketch a possible graph of \( f \).
8. Find all possible antiderivatives of the following.

(a) \( g'(t) = e^t + t^6 + e^{5t} \)  
\[ g(t) = e^t + \frac{t^6}{6} + \frac{e^{5t}}{5} + C \]

(b) \( h'(r) = 3\sin(2r) + \frac{\sqrt{r}}{r} \)  
\[ h(r) = -\frac{3\cos(2r)}{2} + \frac{3}{4} r^{\frac{4}{3}} + C \]

9. Evaluate the following limits.

(a) \[ \lim_{x \to \infty} \frac{x^2}{\ln x} = \frac{\infty}{\infty} = \lim_{x \to \infty} \frac{2x}{\frac{1}{x}} = \lim_{x \to \infty} 2x^2 = \infty \]
\( \text{L'Hôpital's Rule} \)

(b) \[ \lim_{x \to 0} \frac{\sin(12x) - 12x}{x^3} = \frac{0}{0} = \lim_{x \to 0} \frac{12\cos(12x) - 12}{3x^2} = \frac{0}{0} = \lim_{x \to 0} -128 \cos(12x) = -1728 \]
\[ \text{L'Hôpital's Rule} \]

(c) \[ \lim_{x \to 0} \frac{e^x - 1}{\cos x} = \frac{0}{1} = 0 \]

(d) \[ \lim_{x \to 5} \frac{35 - 7x}{2x - 10} = \frac{0}{0} = \lim_{x \to 5} \frac{-7}{2} = -\frac{7}{2} \]
\[ \text{Rewrite as fraction so can use L'Hôpital's Rule} \]

(e) \[ \lim_{x \to 0^+} x^3 \ln x = \lim_{x \to 0^+} x \ln x = -\infty = \lim_{x \to 0^+} \frac{x^3}{-3x^4} \]
\[ \text{L'Hôpital's Rule} \]

(f) \[ \lim_{x \to 0^-} \frac{1}{-x} = -\infty \]

(g) \[ \lim_{x \to 0^+} \frac{1}{x} = \text{undefined} \]

For (f) and (g):