Name:_____

Exam 3- Take-Home Portion

Show all your work to receive full credit for a problem.

Attach this sheet to the solutions you hand in. Even if you attempt the problems in any order, write the solutions in the chronological order.

Please sign and date the following statement:

I declare that the work I am submitting is entirely my own and that I did not confer with anyone (except maybe the instructor) in completing this exam. Further, I declare that I did not use any sources other than my class notes and the course textbook.

Signature: _____

Date: _____

1. (12 pts) Evaluate the following line integrals. (NOTE: There may be more than one way to figure out the answer).

   (a)
   $$\int_C \frac{2x}{y^2+1}dx - \frac{2y(x^2+1)}{(y^2+1)^2}dy$$
   where C is parametrized by $x = t^3 - 1, y = t^6 - t, 0 \leq t \leq 1$.

   (b)
   $$\oint_C (x+z)dx + xdy + ydz$$
   where C is the upper half of the circle $x^2 + z^2 = 9$ in the plane $y = 0$, together with the x-axis from (3,0,0) to (-3,0,0), oriented counterclockwise when viewed from the positive y-axis.

2. Suppose S is the solid given by
   $$S = \{(x, y, z) | x^2 + y^2 + 1 \leq z \leq 5\}$$
   and consider the vector field $\mathbf{F} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$. Assume that ∂S is oriented with an outward normal.

   (a) (6 pts) Compute the surface area of ∂S.

(b) (6 pts) Compute

$$\iint_{\partial S} \mathbf{F} \cdot \mathbf{n} d\sigma$$

directly (only using the definition).

(c) (6 pts) Now compute the integral in part (b) using the divergence theorem. Which of these computations was easier?

(d) (6 pts) Give a physical interpretation for the answer you got for parts (b) and (c).