

Math 206 Section A

Take-home Exam

50 points

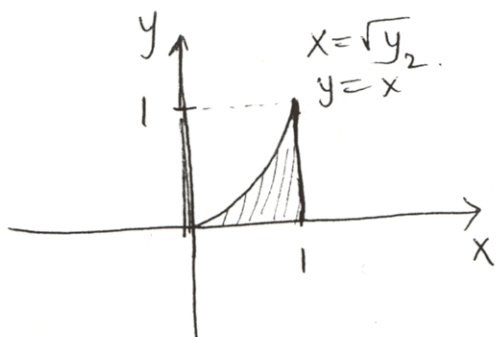
Name: Solutions

Show all your work to receive full credit for a problem.

Attach this sheet to the solutions you hand in. Even if you attempt the problems in any order, write the solutions in the chronological order.

1. (8 points) Sketch the region of integration and evaluate the following integral. If necessary, switch the order of integration. Switch to polar coordinates if needed. Do not use numerical methods to evaluate the integral.

$$\int_0^1 \int_{\sqrt{y}}^1 \sqrt{2+x^3} dx dy.$$



Need to switch order of integration because we cannot evaluate the integral in the given order.

$$\int_0^1 \int_{\sqrt{y}}^1 \sqrt{2+x^3} dx dy = \int_0^1 \int_0^{x^2} \sqrt{2+x^3} dy dx$$

$$\int_0^{x^2} \sqrt{2+x^3} dy = \sqrt{2+x^3} y \Big|_0^{x^2} = \sqrt{2+x^3} x^2.$$

$$\int_0^1 \sqrt{2+x^3} x^2 dx = \frac{1}{3} \cdot \frac{2}{3} (2+x^3)^{3/2} \Big|_0^1 \quad \left(\text{Use substitution } w=2+x^3, dw=3x^2 dx \right)$$

$$= \boxed{\frac{2}{9} (3^{3/2} - 2^{3/2})}$$