Math 206 Section A

Take-home Exam

50 points

Name: _____

Show all your work to receive full credit for a problem.

Attach this sheet to the solutions you hand in. Even if you attempt the problems in any order, write the solutions in the chronological order.

1. **(8 points)** Sketch the region of integration and evaluate the following integral. If necessary, switch the order of integration. Switch to polar coordinates if needed. Do not use numerical methods to evaluate the integral.

$$\int_0^1 \int_{\sqrt{y}}^1 \sqrt{2+x^3}\, dx\, dy\,.$$

2. **(9 points)** Let $f(x, y, z) = x^2 + y^2 + z^2$ and let S be the solid bounded below by the cone $z = \sqrt{x^2 + y^2}$ and above by the sphere $x^2 + y^2 + z^2 = 1$. Write $\int\int\int_S f(x, y, z)\, dV$ as an iterated integral in Cartesian coordinates, spherical coordinates and cylindrical coordinates. Do not evaluate any of the three integrals.

3. **(8 points)** Evaluate the integral $\int\int_M \vec{F} \cdot \vec{n}\, d\sigma$, where $\vec{F} = (x, y, z)$ and M is the part of the surface $z = 25 - x^2 - y^2$ that lies above the xy-plane and it is oriented upward.

4. **(8 points)** Find the mass of the sphere $x^2 + y^2 + z^2 = 4$ if the per-unit-area density at each point of the sphere is given by $g(x, y, z) = \sqrt{4 - x^2 - y^2}$.

5. **(9 points)** Let $\vec{F} = (y, -x)$ and let M be the disk in the xy-plane given by $x^2 + y^2 \leq 1$ oriented upward. Let C be the circle of radius 1 in the xy-plane centered at the origin and oriented counterclockwise.

   (a) Evaluate $\int_C \vec{F} \cdot d\vec{x}$.

   (b) Evaluate $\int\int_M \operatorname{curl} \vec{F} \cdot \vec{n}\, d\sigma$.

   (c) How do the answers in parts (a) and (b) compare? How are C and M related?

6. **(8 points)** Let $f(x, y) = y - x^2$.

   (a) Find grad f. Sketch grad f with Maple.

   (b) On the sketch from part (a), draw the level curves of f for $c = -1, 2$.

   (c) Draw a curve C_1 (on the same sketch) that begins on the level curve for -1 and ends on the level curve for 2. Evaluate $\int_{C_1} \text{grad } f \cdot d\vec{x}$.

   (d) Draw a curve C_2 (on the same sketch) such that $\int_{C_2} \text{grad } f \cdot d\vec{x} = 0$. Explain briefly why the line integral is zero.