NAME____

I___II___III___IV___V___VI___VII___VIII___ IX___X___XI___TOTAL ___________

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Mathematics 205
Linear Algebra
Examination #3

Mr. Haines

(10) I. Suppose $\mathbf{v_1} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and $\mathbf{v_2} = \begin{bmatrix} 1 \\ 1.5 \end{bmatrix}$ and W is the subsspace of $\Re^2$ consisting of all linear combinations of $\mathbf{v_1}$ and $\mathbf{v_2}$.

A. What is the dimension of W ?

B. Give a basis for W .

(15) II. If $T: \Re^2 \rightarrow \Re^2$ reflects points through the line $x_1 = x_2$,

A. What are the eigenvalues of the standard matrix of the linear transformation T?

B. What are the bases for the eigenspaces of the standard matrix of the linear transformation T?

C. Explain what your answer to B means geometrically.

(10) III. Matrix A is row equivalent to matrix B, where

$$A = \begin{bmatrix} -2 & -5 & 8 & 0 & -17 \\ 1 & 3 & -5 & 1 & 5 \\ 3 & 11 & -19 & 7 & 1 \\ 1 & 7 & -13 & 5 & -3 \end{bmatrix} \quad \text{and } B = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & -2 & 0 & 3 \\ 0 & 0 & 0 & 1 & -5 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

A. Give a basis for Col A

B. Give a basis for Nul A

(5) IV. Calculate the dimension of the vector space $\{ (a, b, c, d, e) : a + c = 0 \}$.

(10) V. Suppose $\mathcal{B} = \{\mathbf{b_1}, \mathbf{b_2}\}$ with $\mathbf{b_1} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $\mathbf{b_2} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $A = \begin{bmatrix} 3 & 4 \\ -1 & -1 \end{bmatrix}$.

If $T(\mathbf{x}) = A\mathbf{x}$ for all $\mathbf{x}$ in $\Re^2$,

A. Find the $\mathcal{B}$-matrix for T. Hint: It is <u>not</u> $\begin{bmatrix} 3 & 7 \\ -1 & -2 \end{bmatrix}$.

B. Use your matrix from part A, or some other method, to find $[T(\mathbf{x})]_{\mathcal{B}}$ for $\mathbf{x} = 2\mathbf{b_1} + 3\mathbf{b_2}$.

(10) VI. If A is a 15 x 9 matrix and dim (Nul A) = 2,

A. dim (Row A) = ______________________

B. dim (Col A) = ______________________

C. rank A = ______________________

(10) VII. Suppose $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$.

A. Find the characteristic polynomial of A .

B. Find the two eigenvalues of A .

C. Find a basis for each of the two eigenspaces of A .

(10) VIII. Give an example of

A. a matrix that is diagonalizable but not invertible.

B. a stochastic matrix.

(5) IX. Suppose that A is diagonalizable, which means that there is an invertible matrix P and a diagonal matrix D for which $A = PDP^{-1}$. If $D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -4 \end{bmatrix}$, calculate the determinant of A.

(10) X. Suppose $A = \begin{bmatrix} 0 & 5 \\ -5 & 0 \end{bmatrix}$.

A. Find the eigenvalues of A .

B. Find a basis for each eigenspace in $\mathbb{C}^2$.

C. Express A as the matrix product SR, where S is a scaling matrix and R is a rotation matrix.

(5) XI. TRUE OR FALSE? (Don't guess! The number of incorrect responses will be subtracted from the number of correct ones. Thus, random guessing earns you no points at all.)

________ 1. If a matrix is diagonalizable, then it has an inverse.

________ 2. The kernel of a linear transformation is a vector space.

________ 3. The vector space P_3 of all third degree polynomials is isomorphic to the vector space $\Re^3$.

________ 4. $\Re^2$ is a subspace of $\Re^3$.

_______ 5. The column space of a 4 x 5 matrix is a subspace of $\Re^5$.