Math 105 - Quiz 13 - November 29, 2007

Instructions: Show all of your work and circle your final answers. Calculators are allowed, but notes and books are not.

1. (10 pts.)
   (a) What does the mean value theorem (MVT) say about the function $f(x) = x^2$ on the interval $[1, 2]$? Since $f(x)$ is a polynomial, it is continuous on $(-\infty, \infty)$ and its derivative, $f'(x) = 2x$, is defined everywhere, so $f(x)$ is also differentiable everywhere. MVT says that there is $c$ in $[1, 2]$ such that $f'(c) = \frac{f(2) - f(1)}{2 - 1}$.

   (b) Find all suitable values of $c$.

   
   $f'(c) = 2c$, so 
   
   $2c = \frac{f(2) - f(1)}{2 - 1} = \frac{2^2 - 1^2}{2 - 1} = \frac{3}{1} = 3$.

   
   \[
   2c = 3, \quad \therefore c = \frac{3}{2}.
   \]

2. (10 pts.) Let $f(x) = \frac{1}{x}$. Although $f(-1) = -1$ and $f(1) = 1$, there is no value of $c$ for which $f(c) = 0$. Why doesn’t this contradict the intermediate value theorem (IVT)?

   $f(x) = \frac{1}{x}$ is not continuous at $x = 0$, and $0$ is in $[-1, 1]$, so the IVT doesn’t apply.