NAME:
Show ALL your work CAREFULLY.

(a) Use the Ratio Test to determine whether the following infinite series converges or diverges.

\[
\sum_{m=1}^{\infty} \frac{m!}{(2m)!}
\]

Consider the limit

\[
\lim_{n \to \infty} \frac{a_{n+1}}{a_n} = \lim_{n \to \infty} \frac{(n+1)!}{n!} \cdot \frac{(2n)!}{n!} = \lim_{n \to \infty} \frac{(n+1)!}{n!} \cdot \frac{(2n)!}{(2n+2)(2n+1) \cdot 2n!} = \lim_{n \to \infty} \frac{1}{2} \left( \frac{n+1}{n} \right)^3 = \frac{1}{2} < 1.
\]

By the Ratio Test, the series converges.

(b) Use the Ratio Test and the Alternating Series Test to determine whether the following alternating series converges absolutely or conditionally, or diverges. If it converges, find upper and lower bounds for its limit.

\[
\sum_{n=0}^{\infty} (-1)^n \frac{n^3}{2^n}
\]

We use the Ratio Test to determine if the series converges absolutely. Consider the limit

\[
\lim_{n \to \infty} \frac{|a_{n+1}|}{|a_n|} = \lim_{n \to \infty} \frac{(n+1)^3}{2^{n+1}} \cdot \frac{2^n}{n^3} = \lim_{n \to \infty} \frac{1}{2} \left( \frac{n+1}{n} \right)^3 = \frac{1}{2} < 1.
\]

By the Ratio Test, the series converges absolutely. (The convergence of the series (not absolute) can be deduced by using the Alternating Series Test.) Since the series is alternating, the limit \( S = \sum_{n=0}^{\infty} (-1)^n \frac{n^3}{2^n} \) lies between any two consecutive partial sums. In particular, \( S \) lies between \( 0 = S_0 \) and \( -\frac{1}{2} = S_1 \), i.e., \(-\frac{1}{2} \leq S \leq 0\).

Date: November 28, 2005.