1. Let $A = \begin{bmatrix} 6 & 0 & 1 \\ -2 & 4 & -1 \\ -2 & 0 & 3 \end{bmatrix}$; then A is diagonalizable and has only two eigenvalues, 4 and 5. Find matrices P and D which represent a diagonalization of A. Even though only one of them is used in the diagonalization, find *both* P^{-1} and D^{-1}.

You may find it useful that $\begin{bmatrix} 2 & 0 & 1 \\ -2 & 0 & -1 \\ -2 & 0 & -1 \end{bmatrix}$ is row equivalent to $\begin{bmatrix} 1 & 0 & 1/2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, and $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$ is row equivalent to $\begin{bmatrix} 1 & 0 & 1 \\ -2 & -1 & -1 \\ -2 & 0 & -2 \end{bmatrix}$

Show all your work.

$P=$ ☐ $P^{-1}=$ ☐ $D=$ ☐ $D^{-1}=$ ☐

2. Find a 2×2 matrix A that has $\mathbf{v} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$ and $\mathbf{w} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$ as eigenvectors with associated eigenvalues 3 and 1, respectively. Show all your work, including any matrices you create in order to do this problem. CIRCLE your final matrix A.

3. Mark each statement TRUE if it is always true, and FALSE if there are counter examples. Let A be an $n \times n$ square matrix.

a. A^{-1} exists $\Rightarrow$ A is diagonalizable. ☐

b. A^{-1} exists $\Rightarrow$ A does not have 0 as an eigenvalue. ☐

c. A is diagonalizable $\Rightarrow$ A^{-1} exists. ☐

d. If 0 is not an eigenvalue of A then A is not singular. ☐