Name:______________________________

Mathematics 205
Exam II
November 19, 2010

Problem	Possible	Actual
1	15	
2	20	
3	15	
4	15	
5	20	
6	15 (5)	
Total	100	

You must show all work to receive credit.
No electronic devices other than calculators are permitted.
Give exact answers (such as $\ln 5$ or e^2) unless requested otherwise.

1. Let $A = \begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix}$.

(a) Find the eigenvalues of A.

(b) Find the eigenvectors of A.

(c) What value(s) of k allows A to be diagonalizable?

2. Recall U is an orthogonal matrix if and only if $U^T U = I$.

   (a) Show that if A and B are orthogonal matrices, then AB is an orthogonal matrix.

   (b) Show that if C is an orthogonal matrix, then $\det(C)$ equals 1 or -1.

3. A certain experiment produces the data $(1, 7.9)$, $(2, 5.4)$, and $(3, -9)$. Describe the model that produces a least-squares fit of these points by a function of the form

$$y = A\cos x + B\sin x.$$

4. Let $\vec{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 3 \\ 4 \end{bmatrix}$ and $\vec{v}_2 = \begin{bmatrix} 0 \\ 2 \\ 3 \\ -1 \end{bmatrix}$.

(a) Are $\vec{v}_1$ and $\vec{v}_2$ orthogonal?

(b) Find vectors $\vec{u}_1$ and $\vec{u}_2$ that are in the direction of $\vec{v}_1$ and $\vec{v}_2$ and have norm 1.

(c) Project $\vec{v}_1$ onto $\vec{v}_2$. What is the "error" vector $\vec{z}$?

5. Let $\mathcal{F}(\mathbb{R})$ be the set of functions on the real line. You may use the fact that this set is a vector space in this problem.

   (a) Let $\mathcal{B}(\mathbb{R}) = \{f \in \mathcal{F}(\mathbb{R})$ such that $f(x) < C$ for all $x \in \mathbb{R}$ and for some constant $C\}$. This is the set of functions that are bounded above.

      i. State three elements of $\mathcal{B}(\mathbb{R})$. You may give formula for functions or graphs of functions.

      ii. Is $\mathcal{B}(\mathbb{R})$ a vector space? Justify your answer.

   (b) Let $\mathcal{L}(\mathbb{R}) = \{f \in \mathcal{F}(\mathbb{R})$ such that $\lim_{x \to \infty} f(x) = 0\}$.

      i. State three elements of $\mathcal{L}(\mathbb{R})$. You may give formula for functions or graphs of functions.

      ii. Is $\mathcal{L}(\mathbb{R})$ a vector space? Justify your answer.

6. Let $\mathbb{P}_4$ be the set of polynomials of degree at most 4 and $\mathbb{P}$ be the set of polynomials. You may use the fact that these sets are vector spaces.

   (a) Show that the map $T : \mathbb{P}_4 \to \mathbb{P}_4$ defined by $T(f) = tf'(t)$ is a linear transformation.

   (b) What are the eigenvalues of this transformation? (If you need to, try constructing the matrix of the transformation: Let the polynomial 1 correspond to the vector $\vec{e}_1$ in $\mathbb{R}^5$. What is $T(1)$? Let the polynomial t correspond to $\vec{e}_2$ in $\mathbb{R}^5$. What is $T(t)$? Etc. Once you have constructed the matrix of transformation, find the eigenvalues of that matrix.)

   (c) What are the eigenvectors of this transformation?

   (d) **Bonus:** Suppose T acts on $\mathbb{P}$. What are the eigenvalues and eigenvectors?