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Mathematics 206a
Multivariable Calculus
Examination #3

Mr. Haines

(20) I. Let **a** = (0, 0, 0) , **b** = (0, 3, 6), and C be the straight line segment connecting **a** to **b**.

A. Give a parametrization for C.

B. If $f(x,y,z) = x^2y^3 + x^2y^3 - z - 3y$, compute the line integral of f over C.

C. If $\boldsymbol{F}(x,y,z) = (0,0,y)$, compute the line integral of $\boldsymbol{F}$ over C.

(21) II. Let M be the triangular surface in the plane $x + y + z = 1$ that is cut off by the three coordinate planes. (M lies in the first octant, where $x \geq 0, y \geq 0,$ and $z \geq 0$.)

A. Give a parametrization for the surface M.

B. Set up and evaluate an iterated integral that gives the area of M.

C. Set up and evaluate an iterated integral that gives the surface integral of the vector field **F** (x, y, z) = (0, 0, y) over the surface M.

(10) III. Use The Fundamental Theorem of Line Integrals to evaluate the line integral of $\mathbf{F}(x, y, z) = (2x, 3y^2, 4z^3)$ over a path connecting (0, 0, 0) to (3, 2, 1).

(10) IV. Evaluate the line integral $\oint_C - y^2 dx + xydy$, where C is the boundary of the square cut from the first quadrant by the lines x = 1, y = 1, the x-axis, and the y-axis.

(8) V. Set up but **do not evaluate** an iterated integral to compute the double integral $\iint_R (x^2 + y)\,dA$, where R is the right triangle with vertices (0, 0), (0, 2), and (1, 0)

(10) VI. Suppose **f** is the path defined by $\mathbf{f}(t) = (\sqrt{t}, t, t^2)$ for positive numbers t.
Set up, but **do not evaluate**, an integral whose value is the arc length of **f** between (1, 1, 1) and (2, 4, 16).

(21) VII. Let **F** (x, y, z) = $(x+3z, 2y, 3x+5z)$.

A. Prove that **F** is path independent on $\Re^3$.

B Find a function whose gradient is **F** .

C If C is a path connecting (0, 0, 0) to (2, 2, 2), calculate the path integral

$$\int_C \mathbf{F} \bullet d\mathbf{x}.$$