

TEST 2B

Math 205
11/18/13

Name: _____
by writing my name I swear by the honor code

Read all of the following information before starting the exam:

- Show all work, clearly and in order if you want to get full credit (matrices can and should be reduced into RREF with calculator without showing steps unless otherwise indicated). I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Circle or otherwise indicate your final answers.
- Please keep your written answers brief; be clear and to the point. I will take points off for rambling and for incorrect or irrelevant statements.
- This test has 7 problems and one Bonus and is worth 95 points, It is your responsibility to make sure that you have all of the pages!
- Good luck!

1. (20 points) Consider $A = \begin{bmatrix} 3 & 5 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 0 & 0 \\ 2 & -1 & -2 \\ -1 & 2 & 4 \end{bmatrix}$

a. (4 pts) Find the characteristic equation $c_A(x)$ for A . Find the eigenvalues of A .

b. (4 pts) Find the characteristic equation $c_B(x)$ for B . Find the eigenvalues of B .

c. (5 pts) Determine whether A is diagonalizable. If so, find P and D such that $A = PDP^{-1}$.

d. (5 pts) Determine whether B is diagonalizable. If so, find P and D such that $B = PDP^{-1}$.

e. (2 pts) What is $c_A(A)$ and what is $c_B(B)$? (You should NOT have to calculate this to answer this question).

2. (15 points) Given $A = \begin{bmatrix} 0 & 1 & -2 & 2 & 0 \\ -1 & 3 & 0 & 1 & 6 \\ 8 & -1 & 3 & 5 & 1 \end{bmatrix} \xrightarrow{rref} \begin{bmatrix} 1 & 0 & 0 & 1 & -6 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & -1 & 1 \end{bmatrix}$. Find a **basis** for the following.

a. (5 pts) $\text{Col}(A)$

b. (5 pts) $\text{Nul}(A)$

c. (5 pts) $\text{Row}(A)$

3. (9 points) Given an $m \times n$ matrix A . Prove (verify in generality) that $\text{Nul}(A)$ is a **subspace** of \mathbb{R}^n .

4. (10 points)

- ___ If A is a 2×2 matrix and has 2 distinct eigenvalues, then A must be diagonalizable.
- ___ If matrices A and B have the same reduced row echelon form then $\text{Row}(A) = \text{Row}(B)$.
- ___ If H is a subspace of \mathbb{R}^3 , then there is a 3×3 matrix A such that $H = \text{Col}(A)$.
- ___ A subspace is also a vectorspace.
- ___ A linearly independent set in a subspace H is a basis for H .

- 5.** (8 points) Consider vectors of the form $\begin{bmatrix} a - 2b + 5c \\ 2a + 5b - 8c \\ -a - 4b + 7c \\ 3a + b + c \end{bmatrix}$.
- a. (3 pts) Show that the vectors form a subspace of \mathbb{R}^4 .

- b. (5 pts) Find a basis for the set.

6. (15 points) Short answer.

a. (5 pts) Prove that if 0 is an eigenvalue of A , then A is not invertible (or an equivalent statement opposite to the IMT).

b. (5 pts) Let A be an $m \times n$ matrix, and let B be an $n \times p$ matrix such that $AB = 0$. Show that $\text{rank}(A) + \text{rank}(B) \leq n$ [Hint: one of the four subspaces $\text{Nul}(A)$, $\text{Col}(A)$, $\text{Nul}(B)$, $\text{Col}(B)$ is contained in one of the other three spaces.]

c. (5 pts) Use coordinate vectors to test whether the set of polynomials span \mathbb{P}_2 .

$$\{1 - 3t + 5t^2, -3 + 5t - 7t^2, -4 + 5t - 6t^2, 1 - t^2\}$$

- 7.** (18 points) Let $\beta = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$ be a basis for $M_{2 \times 2}(\mathbb{R})$.
- a.** (7 pts) Use coordinate vectors to prove whether $\left\{ \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix} \right\}$ is a basis for $M_{2 \times 2}(\mathbb{R})$.

Consider the transformation $T : M_{2 \times 2}(\mathbb{R}) \rightarrow M_{2 \times 2}(\mathbb{R})$ where $T(A) = -A^T$. Recall an element in $M_{2 \times 2}(\mathbb{R})$ can be represented as $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$.

- b.** (3 pts) What is the $\text{Ker}(T)$?

The transformation T has $\lambda = 1$ and $\lambda = -1$ as eigenvalues. (ie. There exists vectors \vec{x} such that $T(x) = \vec{x}$ or $T(\vec{x}) = -\vec{x}$.)

c. (4 pts) Without finding the matrix associated with T find the matrices in the eigenspace associated with $\lambda = 1$, then find a basis for the eigenspace.

d. (4 pts) Without finding the matrix associated with T , find the matrices in the eigenspace associated with $\lambda = -1$, then find a basis for the eigenspace.

Bonus Question (2 Extra Credit Points):

The **left principal minors** of a matrix are the determinants of the matrices formed by deleting the last row and the last column of a matrix (creating a 3×3 matrix), then the last two columns and last two rows of the original matrix (creating a 2×2 matrix), then the last three rows and last three columns of the original matrix until the matrix is just 1×1 (a scalar).

A matrix is called positive definite if the determinant of the matrix and all the principal minors are positive.

Determine whether A is positive definite.

$$A = \begin{bmatrix} 6 & 1 & 0 & -1 \\ 2 & 2 & 0 & 1 \\ 0 & -3 & 8 & 0 \\ 0 & 1 & 0 & 5 \end{bmatrix}$$

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