TEST 2A

Math 205
11/18/13

Name: ______________________
by writing my name I swear by the honor code

Read all of the following information before starting the exam:

- Show all work, clearly and in order if you want to get full credit (matrices can and should be reduced into RREF with calculator without showing steps unless otherwise indicated). I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Circle or otherwise indicate your final answers.
- Please keep your written answers brief; be clear and to the point. I will take points off for rambling and for incorrect or irrelevant statements.
- This test has 7 problems and one Bonus and is worth 95 points, It is your responsibility to make sure that you have all of the pages!
- Good luck!

1. (*15 points*) Given $A = \begin{bmatrix} 0 & 1 & -2 & 2 & 0 \\ -1 & 3 & 0 & 1 & 6 \\ 8 & -1 & 3 & 5 & 1 \end{bmatrix} \rightarrow_{rref} \begin{bmatrix} 1 & 0 & 0 & 1 & -6 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & -1 & 1 \end{bmatrix}$. Find a **basis** for the following.

a. (*5 pts*) Col(A)

b. (*5 pts*) Nul(A)

c. (*5 pts*) Row(A)

2. (*9 points*) Given an $m \times n$ matrix A. Prove (verify in generality) that Nul(A) is a **subspace** of $\mathbb{R}^n$.

3. (*10 points*)

- ____ If A is a 2×2 matrix and has 2 distinct eigenvalues, then A must be diagonalizable.
- ____If matrices A and B have the same reduced row echelon form then Row(A) = Row(B).
- ____ If H is a subspace of $\mathbb{R}^3$, then there is a 3×3 matrix A such that H = Col(A).
- ____ A subspace is also a vectorspace.
- ____ A linearly independent set in a subspace H is a basis for H.

4. (*8 points*) Consider vectors of the form $\begin{bmatrix} a-2b+5c \\ 2a+5b-8c \\ -a-4b+7c \\ 3a+b+c \end{bmatrix}$.

a. (*3 pts*) Show that the vectors form a subspace of $\mathbb{R}^4$.

b. (*5 pts*) Find a basis for the set.

5. (*20 points*) Consider $A = \begin{bmatrix} 3 & 5 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 0 & 0 \\ 2 & -1 & -2 \\ -1 & 2 & 4 \end{bmatrix}$

a. (*4 pts*) Find the characteristic equation $c_A(x)$ for A. Find the eigenvalues of A.

b. (*4 pts*) Find the characteristic equation $c_B(x)$ for B. Find the eigenvalues of B.

c. (*5 pts*) Determine whether A is diagonalizable. If so, find P and D such that $A = PDP^{-1}$.

d. (*5 pts*) Determine whether B is diagonalizable. If so, find P and D such that $B = PDP^{-1}$.

e. (*2 pts*) What is $c_A(A)$ and what is $c_B(B)$? (You should NOT have to calculate this to answer this question).

6. (*18 points*) Let $\beta = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$ be a basis for $M_{2\times 2}(\mathbb{R})$.

a. (*7 pts*) Use coordinate vectors to prove whether $\left\{ \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix} \right\}$ is a basis for $M_{2\times 2}(\mathbb{R})$.

Consider the transformation $T : M_{2\times 2}(\mathbb{R}) \to M_{2\times 2}(\mathbb{R})$ where $T(A) = -A^T$. Recall an element in $M_{2\times 2}(\mathbb{R})$ can be represented as $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$.

b. (*3 pts*) What is the $\text{Ker}(T)$?

The transformation T has $\lambda = 1$ and $\lambda = -1$ as eigenvalues. (ie. There exists vectors $\vec{x}$ such that $T(x) = \vec{x}$ or $T(\vec{x}) = -\vec{x}$.)

c. (*4 pts*) Without finding the matrix associated with T find the matrices in the eigenspace associated with $\lambda = 1$, then find a basis for the eigenspace.

d. (*4 pts*) Without finding the matrix associated with T, find the matrices in the eigenspace associated with $\lambda = -1$, then find a basis for the eigenspace.

7. (*15 points*) Short answer.

a. (*5 pts*) Prove that if 0 is an eigenvalue of A, then A is not invertible.

b. (*5 pts*) Let A be an $m \times n$ matrix, and let B be an $n \times p$ matrix such that $AB = 0$. Show that rank (A) + rank$(B) \leq n$ [Hint: one of the four subspaces Nul(A), Col(A), Nul(B), Col(B) is contained in one of the other three spaces.)

c. (*5 pts*) Use coordinate vectors to test the whether the set of polynomials span $\mathbb{P}_2$.

$$\{1 - 3t + 5t^2, -3 + 5t - 7t^2, -4 + 5t - 6t^2, 1 - t^2\}$$

Bonus Question (2 Extra Credit Points):

The **left principal minors** of a matrix are the determinants of the matrices formed by deleting the last row and the last column of a matrix (creating a 3 $\times$ matrix), then the last two columns and last two rows of the original matrix (creating a 2×2 matrix), then the last three rows and last three columns of the original matrix until the matrix is just 1 $\times$ 1 (a scalar).
A matrix is called positive definite of the determinant of the matrix and all the principal minors are positive.
Determine whether A is positive definite.

$$A = \begin{bmatrix} 6 & 1 & 0 & -1 \\ 2 & 2 & 0 & 1 \\ 0 & -3 & 8 & 0 \\ 0 & 1 & 0 & 5 \end{bmatrix}$$

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