TEST 2

Math 205
11/18/11

Name: ______________________
by writing my name i swear by the honor code

Read all of the following information before starting the exam:

- Show all work, clearly and in order if you want to get full credit (matrices should be reduced into RREF with calculator and you can just show the output). I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Circle or otherwise indicate your final answers.
- Please keep your written answers brief; be clear and to the point. I will take points off for rambling and for incorrect or irrelevant statements.
- This test has 7 problems and is worth 100 points, It is your responsibility to make sure that you have all of the pages!
- Good luck!

1. (*15 points*) Given $A = \begin{bmatrix} 1 & 2 & 1 & 1 & 5 \\ -2 & -4 & 0 & 4 & -2 \\ 1 & 2 & 2 & 4 & 9 \end{bmatrix}$. Find a **basis** for the following.

a. (*5 pts*) Col(A)

b. (*5 pts*) Nul(A)

c. (*5 pts*) Row(A)

2. (*9 points*) Prove (verify in generality) that H = span$\{\vec{v_1}, \vec{v_2}, \ldots, \vec{v_p}\}$ where each $\vec{v_i}$ is in V is a subspace of V.

3. (*12 points*) $A = \begin{bmatrix} 2 & k \\ 0 & 2 \end{bmatrix}$

a. (*3 pts*) What is the det(A)?
b. (*3 pts*) For what k is A invertible?
c. (*3 pts*) What are the eigenvalues of A?
d. (*3 pts*) For what values of k will A be diagonalizable?

4. (*13 points*) Determine if the matrix A can be diagonalized. If so, find P and D such that $A = PDP^{-1}$. The characteristic equation for A is $(\lambda+1)^2(\lambda-2) = 0$. (Be careful with your arithmetic).

$$A = \begin{bmatrix} -4 & -3 & -3 \\ 0 & -1 & 0 \\ 6 & 6 & 5 \end{bmatrix}$$

5. (*11 points*)

a. (*5 pts*) Let $[\vec{x}]_\beta = \begin{bmatrix} \ell \\ v \\ \ell \end{bmatrix}$ be the coordinate vector of $\vec{x}$ with respect to the basis, $\beta = \{\vec{O}, \vec{E}, \vec{A}\}$.

Determine $\vec{x}$.

b. (*6 pts*) Let $H = \{\begin{bmatrix} a & 0 \\ b & c \end{bmatrix} \mid a, b, c \in \mathbb{R}\}$. Show H is isomorphic to $\mathbb{R}^3$ by using the fact that the mapping $\vec{x} \mapsto [\vec{x}]_\beta$ is an isomorphism between H and $\mathbb{R}^3$ when you define a basis β of H and $[\vec{x}]_\beta$ in $\mathbb{R}^3$.

6. (*13 points*) Let A be an invertible, diagonalizable matrix ($A = PDP^{-1}$) where

$$P = \begin{bmatrix} a & b \\ c & d \end{bmatrix},\ ad - bc = 1 \text{ and } D = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}.$$

Find the general form for A^{-k} (Remark: A^{-k} is just $(A^{-1})^k$).

7. (*27 points*) Short answer.

a. (*6 pts*) A matrix is called **full rank** when its rank is as large as possible. Let A be an $m \times n$ matrix of full rank. Justify whether or not $A\vec{x} = \vec{b}$ will be consistent if:

- $m = n$
- $m > n$
- $m < n$

b. (*4 pts*) Give an example of a subset H of some vector space V of your choosing that is NOT a subspace. Briefly explain why.

c. (*4 pts*) A is a 6×6 matrix with 4 eigenvalues. Two of the eigenspaces are two-dimensional. Is A always diagonalizable? Why or why not?

d. (*5 pts*) Suppose A is an $n \times n$ matrix. If rank(A) = n, what is the Col(A), Row(A), and the Nul(A).

e. (*4 pts*) Let A and B be two matrices where AB exists. Explain why Nul(B) is a subset of Nul(AB) (ie. explain why every vector in Nul(B) is also in Nul(AB).

f. (*4 pts*) Consider the linear transformation which sends a 2×2 matrix to its transpose. Without making the matrix of this transformation find an eigenvalue and describe the associated eigenspace.

8. (*1 point*) Bonus: In problem 6, if $\lambda_1 = \lambda_2$ what is the general form of A^{-k}?

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