MATH 205A,B LINEAR ALGEBRA - PROF. P. WONG

EXAM II - NOVEMBER 16, 2015

NAME: **Section:**(Circle one) A(8 : 00) B(9 : 30)

Instruction: Read each question carefully. Explain **ALL** your work and give reasons to support your answers.

Advice: DON'T spend too much time on a single problem.

Problems	**Maximum Score**	**Your Score**
1.	20	
2.	20	
3.	20	
4.	20	
5.	20	
Total	100	

1. Let

$$A = \begin{bmatrix} 3 & 2 \\ 1 & 0 \end{bmatrix}.$$

(a)(7 pts) Find the eigenvalues of A.

(b)(7 pts) For each of the eigenvalue(s) found in (a), determine the corresponding eigenspaces by giving a basis for each such subspace.

(c)(6 pts) Is A diagonalizable? If so, find an invertible matrix P such that $P^{-1}AP$ is diagonal.

2. Let

$$A = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 3 & 2 & 5 & 1 \\ 0 & 4 & 4 & -4 \end{bmatrix}.$$

(a)(10 pts) Find a basis for the column space Col$A$ of A.

(b)(10 pts) Find a basis for the null space Nul$A$ of A.

3. (a)(5 pts) Let A be a 3×4 matrix. If $\dim \mathrm{Nul} A = 3$, what is the rank of A? Justify your answer.

(b)(5 pts) Suppose B is a 4×4 matrix with eigenvalues $2, 3, -1$ such that the eigenspace corresponding to 2 has dimension 1; the eigenspace corresponding to 3 has dimension 1; and the eigenspace corresponding to -1 has dimension 1. Determine whether B is diagonalizable. Justify your answer.

(c)(5 pts) Let B be the matrix as in (b). Find $\det(B + I)$, the determinant of the matrix $(B + I)$. Justify your answer.

(d)(5 pts) Let B be the matrix as in (b). What is the dimension of $\mathrm{Col}(B - 3I)$? Justify your answer.

4. (a) Let $\mathcal{B} = \{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ be a basis for $\mathbb{R}^3$ where

$$\vec{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} -4 \\ 5 \\ 6 \end{bmatrix}, \vec{v}_3 = \begin{bmatrix} 7 \\ -8 \\ 9 \end{bmatrix}.$$

(i)(4 pts) Find the coordinate matrix $P_{\mathcal{B}}$.

(ii)(8 pts) Suppose $\vec{u} = \begin{bmatrix} 5 \\ -12 \\ 3 \end{bmatrix}$. Find $[\vec{u}]_{\mathcal{B}}$, the $\mathcal{B}$-coordinates of $\vec{u}$.

(b)(8 pts) **Use row reduction** to find the determinant $\det A$ of the following matrix

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 4 & 2 & 3 \\ 1 & 3 & 0 \end{bmatrix}.$$

5. (a)(7 pts) Let $T : \mathbb{P}_2 \to \mathbb{P}_3$ be given by $T(\mathbf{p}(x)) = x\mathbf{p}(x)$. Show that T is a linear transformation.

(b)(3 pts) Find $KerT$, the kernel of T.

(c)(5 pts) Let $\mathbf{p}_1(x) = x + x^3$ be in $\mathbb{P}_3$. Does $\mathbf{p}_1(x)$ lie in the Range of T? Justify your answer.

(d)(5 pts) Let $\mathbf{p}_2(x) = 1 + x$ be in $\mathbb{P}_3$. Does $\mathbf{p}_2(x)$ lie in the Range of T? Justify your answer.