Math 205A Quiz 7, page 1 November 16, 2007 NAME *Suggested Sol'ns*

1. Fact: The matrix $A = \begin{bmatrix} 0 & 1 & 4 & -1 & -2 & -3 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 4 & 0 & -12 & 9 & 34 & 25 \\ 3 & 6 & 15 & 0 & 12 & -3 \\ 2 & 4 & 10 & 0 & 8 & -2 \end{bmatrix}$ is row equivalent to $\begin{bmatrix} 1 & 0 & -3 & 0 & 4 & -5 \\ 0 & 1 & 4 & 0 & 0 & 2 \\ 0 & 0 & 0 & 1 & 2 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

1A. Find each of the following:

i) The rank of A. = dim col space = 3

ii) The dimension of the null space of A. also, 3

iii) A basis for the column space of A.

The pivot columns of A form a basis, as discussed in class ∴ $\left\{ \begin{bmatrix} 0 \\ 0 \\ 4 \\ 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ 6 \\ 4 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 9 \\ 0 \\ 0 \end{bmatrix} \right\}$

BE WARE! the corresponding cols. of the RREF are NOT a basis for Col(A)!

iv) A basis for the row space of A.

The non-zero rows of the RREF form of A are a basis for both it (the RREF) and A also; ie the following is a basis for either:

$\{ [1\ 0\ -3\ 0\ 4\ -5], [0\ 1\ 4\ 0\ 0\ 2], [0\ 0\ 0\ 1\ 2\ 5] \}$

2. Suppose $B \in M_{3\times 5}$.

2A. What are the maximum and minimum possible dimensions for Nul(B)? MAX: 5 MIN: 2

dim (Nul(B)) = # of free variables. If B = the zero matrix, all variables are free; but since there are only 3 rows, at least 2 variables are free.

2B. What are the maximum and minimum possible values for the rank of B? MAX: 3 MIN: 0

rank = # of pivot columns. The minimum # is 0 (if B is all 0's) and the maximum is the # of rows 3.

(note that max (null(B)) + min(rank(B)) = 5
AND min (null(B)) + max (rank(B)) = 5
AND # of columns of B = 5... is this a coincidence?)

3. Let $C = \begin{bmatrix} -4 & 4 \\ -5 & 8 \end{bmatrix}$

3A. Find the characteristic polynomial of C.

it's $\det(C - \lambda I)$

$= \det\left(\begin{bmatrix} -4 & 4 \\ -5 & 8 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \right)$

$= \det\left(\begin{bmatrix} -4-\lambda & 4 \\ -5 & 8-\lambda \end{bmatrix} \right) = \begin{vmatrix} -4-\lambda & 4 \\ -5 & 8-\lambda \end{vmatrix} = (-4-\lambda)(8-\lambda) + 20 = \lambda^2 - 4\lambda - 32 + 20$

$= \lambda^2 - 4\lambda - 12$

3B. Find the eigenvalues of C using the answer to (3A).

Setting $\lambda^2 - 4\lambda - 12 = 0$:

$(\lambda - 6)(\lambda + 2) = 0$ so the eigenvalues are $+6$ and -2.

3C. Find an eigenvector of C corresponfing to the *larger* of the two eigenvalues in (3B).

6 is the larger. We need sol'ns of $C\vec{x} = 6\vec{x}$

or, $\left[\begin{array}{cc|c} -4-6 & 4 & 0 \\ -5 & 8-6 & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} -10 & 4 & 0 \\ -5 & 2 & 0 \end{array}\right]$

$\sim \left[\begin{array}{cc|c} -10 & 4 & 0 \\ 0 & 0 & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & -2/5 & 0 \\ 0 & 0 & 0 \end{array}\right]$ so $x_1 = \frac{2}{5}x_2$ where x_2 is free;

eigenvectors therefore have the form $x_2 \begin{bmatrix} 2/5 \\ 1 \end{bmatrix}$. two specific examples are, say, $\begin{bmatrix} 2/5 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 5 \end{bmatrix}$.