Math 205A Quiz 07 page 1 November 14, 2008 NAME suggested solutions

1. Suppose A is $\begin{bmatrix} 1 & 4 & 2 \\ -7 & 11 & 3 \\ 9 & -5 & 3 \end{bmatrix}$ and let $\mathbf{v} = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$ and $\mathbf{w} = \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}$.

1A. Is $\mathbf{v}$ an eigenvector of A? Explain. If it is, give the eigenvalue.

$A\vec{v} = \begin{bmatrix} 14 \\ 14 \\ 14 \end{bmatrix} = \lambda \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$ where $\lambda = 7$, so yes $\vec{v}$ is an eigenvector of A because $\vec{v} \neq \vec{0}$ and there exists $\lambda \in \mathbb{R}$ s.t. $A\vec{v} = \lambda\vec{v}$.

1B. Is $\mathbf{w}$ an eigenvector of A? Explain. If it is, give the eigenvalue.

$A\vec{w} = \begin{bmatrix} 6 \\ -8 \\ 24 \end{bmatrix}$. Since $\begin{bmatrix} 6 \\ -8 \\ 24 \end{bmatrix} \neq \lambda \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}$ for any $\lambda \in \mathbb{R}$, $\begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}$ is not an eigenvector of A.

1C. Fact: One eigenvalue for A is $\lambda = 4$. Find a basis for the corresponding eigenspace.

This eigenspace is the same as the null space of $(A - 4I) = \begin{bmatrix} -3 & 4 & 2 \\ -7 & 7 & 3 \\ 9 & -5 & -1 \end{bmatrix}$

The latter matrix has RREF $\begin{bmatrix} 1 & 0 & 2/7 \\ 0 & 1 & 5/7 \\ 0 & 0 & 0 \end{bmatrix}$, so a basis for its nullspace is $\boxed{\left\{ \begin{bmatrix} -2/7 \\ -5/7 \\ 1 \end{bmatrix} \right\}}$, or, $\left\{ \begin{bmatrix} -2 \\ -5 \\ 7 \end{bmatrix} \right\}$.

2. Let $C = \begin{bmatrix} 3 & 4 \\ 6 & 7 \end{bmatrix}$.

2a. Find the characteristic polynomial of C.

$$\text{char poly}(C) = \det(C - \lambda I_2) = \begin{vmatrix} 3-\lambda & 4 \\ 6 & 7-\lambda \end{vmatrix} = (3-\lambda)(7-\lambda) - 24 = 21 - 10\lambda + \lambda^2 - 24 = \boxed{\lambda^2 - 10\lambda - 3}$$

2b. Find any real eigenvalues of C or explain why there are none.

$$\lambda = \frac{10 \pm \sqrt{100 - 4(1)(-3)}}{2 \cdot 1} = \frac{10 \pm \sqrt{112}}{2} = \frac{10 \pm \sqrt{16 \cdot 7}}{2} = \frac{10 \pm 4\sqrt{7}}{2} = \boxed{5 \pm 2\sqrt{7}}$$

3. Suppose B and D are 4 by 4 square matrices and Det(B) is 4 and Det(BD) is 8. Find each of the following. Write "Can't Do" if it is not possible to find the answer with the information given.

a. Det(D). $\boxed{2}$ Since $\text{Det}(BD) = \text{Det}(B)\,\text{Det}(D)$, $8 = 4 \cdot \text{Det}(D)$

b. Det(DB). $\boxed{8}$ $\text{Det}(DB) = \text{Det}(D) \cdot \text{Det}(B) = 2 \cdot 4 = 8$ (although $DB \neq BD$ in general)

c. Det(B^T). $\boxed{4}$

d. Det(B^{-1}). $\boxed{1/4}$ if B^{-1} exists, then $1 = \det(I) = \det(BB^{-1}) = \det(B)\det(B^{-1}) = 4 \cdot \frac{1}{4}$

e. Det($3B$). $3^4 \cdot 4 = \boxed{324}$ ($3B$ is obtained from B by multiplying EACH of its 4 rows by 3; each such multiplication affects det by a factor of 3)

f. Det(B^3) $\boxed{64}$ $= \det(B)\det(B)\det(B) = 4 \cdot 4 \cdot 4$

g. Det($B + B$) $\boxed{64}$ $= \text{Det}(2B) = 2^4 \cdot \det(B) = 2^4 \cdot 4 =$

h. Det($B + D$) $\boxed{\text{CAN'T DO}}$