Exam #2, Math 205B (Linear Algebra)

This take-home exam is due on **Friday, November 14** by 5 PM. You may consult the textbook (or any other book) and any class notes and handouts, but **please do not discuss any details of this exam with anyone except me!** Please sign the bottom of this sheet and turn it in with your exam. You may ask me questions about the exam, but I reserve the right to give unsatisfying answers. Matrix multiplications and reduced row echelon forms may be done on MATLAB or a calculator, but please show all other work.

1. (16 points) Find the complete solution of the system $\begin{pmatrix} 1 & 3 & 1 & 2 & 2 \\ 1 & 2 & 1 & 4 & 3 \\ 1 & 4 & 2 & 2 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 2 \end{pmatrix}$.

2. (20 points) Find a basis for each of the four subspaces associated with the matrix

$$A = \begin{pmatrix} 1 & 1 & 3 & 5 \\ 2 & 1 & 2 & 7 \\ 2 & 3 & 10 & 13 \\ 5 & 3 & 7 & 19 \end{pmatrix}$$

What is the factored form of A that displays these bases?

3. (14 points) The vectors $\begin{pmatrix} 1 \\ 1 \\ 1 \\ 2 \end{pmatrix}$, $\begin{pmatrix} 1 \\ 2 \\ 1 \\ 5 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 4 \\ 1 \\ 4 \end{pmatrix}$ span a 3-dimensional subspace of $\mathbb{R}^4$. Find an orthonormal basis for it.

4. (18 points) Let $\vec{v}_1 = \begin{pmatrix} 1 \\ 3 \\ 1 \\ 1 \end{pmatrix}$, $\vec{v}_2 = \begin{pmatrix} 1 \\ 1 \\ 3 \\ 1 \end{pmatrix}$, and $\vec{v}_3 = \begin{pmatrix} 3 \\ 3 \\ 1 \\ -1 \end{pmatrix}$, and let S be the subspace of $\mathbb{R}^4$ spanned by $\vec{v}_1$ and $\vec{v}_2$. Find the matrix P that projects vectors in $\mathbb{R}^4$ onto S, and the matrix R that reflects through S. Find also the projection $\vec{p}$ of $\vec{v}_3$ onto S and the reflection $\vec{r}$ of $\vec{v}_3$ through S.

5. (18 points) Explain how you can tell that $P = \frac{1}{12} \begin{pmatrix} 5 & 2 & -1 & -1 & 2 & 5 \\ 2 & 2 & 2 & 2 & 2 & 2 \\ -1 & 2 & 5 & 5 & 2 & -1 \\ -1 & 2 & 5 & 5 & 2 & -1 \\ 2 & 2 & 2 & 2 & 2 & 2 \\ 5 & 2 & -1 & -1 & 2 & 5 \end{pmatrix}$ is a projection matrix.

Find a basis for the subspace T of $\mathbb{R}^6$ that P projects onto, and a basis for $T^\perp$ (the orthogonal complement of T).

6. (8 points) Continuing a theme from the third homework set, construct a 4×4 matrix whose column space equals its nullspace.

I affirm that I did not receive help from another person in doing this exam, nor did I give help to another student in the class.

(signed) ______________________________