1. A "True or False" question. In this problem, assume A and B are 4×4 square matrices. Suppose:
i) rref(A)= R where R has exactly three leading-1's,
ii) rref(B)= S and $S = I_4$. In the left margin, next to each statement, write "T" if the statement is it always true and "F" if it's not always true, for any such matrices A and B.

a) A is an invertible matrix.

b) The determinant of B is non-zero.

c) The equation $A\mathbf{x} = \mathbf{b}$ has infinitely many solutions $\mathbf{x}$ for each $\mathbf{b} \in \mathbf{R}^4$.

d) Nul(A)=Nul(R).

e) rank(A)=3.

f) Col(B)=Col(S).

g) The number 0 is an eigenvalue for A.

h) The number 0 is an eigenvalue for B.

i) Nul(B)= $\{\mathbf{0}\}$.

j) A and B are not row equivalent.

k) A and B are not similar.

l) In any basis of Nul(A), there is exactly one vector.

2. Let $B = \begin{bmatrix} 5 & 1 & 2 \\ -5 & 11 & 2 \\ 10 & -4 & 2 \end{bmatrix}$ Here are some facts about B:

(i) $\mathbf{v} = \begin{bmatrix} 1 \\ 1 \\ -3 \end{bmatrix}$ is in Nul(B),

(ii) $\lambda = 10$ is an eigenvalue of B,

(iii) $\mathbf{u} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ is an eigenvector for B.

You should be able to answer these questions without finding any determinants and with maybe just one rref.

a) What is the eigenvalue corresponding to the eigenvector $\mathbf{u}$?

b) What is the characteristic polynomial of B in factored form?

c) Show that B is diagonalizable by exhibiting P, D and P^{-1} that have the required properties.

3. Let $C = \begin{bmatrix} 4 & 3 & -1 \\ 0 & 7 & -1 \\ 0 & 6 & 2 \end{bmatrix}$.

a) Find the characteristic polynomial of C in factored form starting from $\det(C - \lambda I)$; show all your work.

b) One of the eigenvalues should have multiplicity 2. Find a basis for the eigenspace of that eigenvalue.

c) Without actually finding P, D and P^{-1}, explain why C is diagonalizable.

4. Let M be the 4×4 matrix here. FACT: The product of

$$\underbrace{\begin{bmatrix} 6 & 1 & 2 & 7 \\ 2 & 6 & 2 & -4 \\ 2 & 3 & 6 & -3 \\ 2 & -4 & 2 & 6 \end{bmatrix}}_{M} \text{ and } \begin{bmatrix} 12 & 7 & 5 & -3 & 1 & 0 & 10 & -2 \\ -1 & 2 & 6 & 2 & 0 & 0 & 12 & -1 \\ 0 & 6 & 7 & 1 & -1 & 0 & 14 & 6 \\ 7 & 0 & 0 & 2 & 0 & 0 & 0 & 3 \end{bmatrix} \text{ is } \begin{bmatrix} 120 & 56 & 50 & 0 & 4 & 0 & 100 & 20 \\ -10 & 38 & 60 & 0 & 0 & 0 & 120 & -10 \\ 0 & 56 & 70 & 0 & -4 & 0 & 140 & 20 \\ 70 & 18 & 0 & 0 & 0 & 0 & 0 & 30 \end{bmatrix}$$

(a) Use this fact to find the eigenvalues of M, and bases for their respective eigenspaces. Note the fact contains useful and *not* useful information!

(b) Is M diagonalizable? (Y/N)

5. Let $D = \begin{bmatrix} a & 0 & 1 \\ c & 2 & 0 \\ 1 & 0 & e \end{bmatrix}$ and suppose det(D)= -10. Find each of the following:

(a) $\det(D^3)$

(b) $\det(3D)$

(c) det (D+D)

(d) $\det(D^{-1})$

(e) $|\ (|D|)\ |$

(f) $|D^T|$

(g) ae

(h) $\det\left(\begin{bmatrix} a & 0 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & e \end{bmatrix}\right)$