Math 206 Section A

Test 2

75 points

Name: ____

Show all your work to receive full credit for a problem.

There are eight questions. Questions are printed on both sides of a page.

1. **(8 points)** Sketch the following set. Determine if the set is open, closed, or neither. Use the definition of open sets and closed sets to illustrate this with your sketch. Also find the boundary and complement.

$$A = \{(x, y, z) \in \mathbb{R}^3 \mid x^2 + y^2 > 4\}$$

2. **(9 points)** Determine the point(s) of discontinuity in the following function. Explain why the point(s) are points of discontinuity. Are the discontinuities removable? Explain.

$$f(x, y) = \frac{3x^2y}{x^4 + y^2}.$$

3. **(10 points)** Find all the critical points of the function $f(x, y) = 9x - x^3 - 4y^2$. Use the second derivative test to determine whether each critical point is a local minimum, a local maximum or a saddle point.

4. **(10 points)** Let $g(x, y) = x^2 - 3y^2$, $f(x, y) = (xy, x + y^2)$, and $\vec{a} = (2, 1)$. Let $h = g \circ f$.

(a) Use the chain rule to write a formula for $Dh(\vec{a})$.

(b) Use $h(\vec{a})$ and $Dh(\vec{a})$ to find an approximation for $h(1.99, 1.01)$.

5. **(9 points)** The quantity of beef, Q (in pounds per week) purchased in a supermarket is a function of the price of beef, b, and the price of chicken, c, (where b and c are in dollars per pound.) So we have $Q = f(b, c)$. Suppose $f_b(1.99, 3.99) = -400$ and $f_c(1.99, 3.99) = 300$. Use this information to answer the following questions.

(a) Explain in words the meaning of the statement $f_b(1.99, 3.99) = -400$.

(b) Find the differential dQ at the point $(1.99, 3.99)$.

(c) Use your answer in part (b) to estimate the change in the quantity of beef purchased in the supermarket if the price of beef increases by \$0.50 per pound and the price of chicken decreases by \$0.50 per pound.

6. **(10 points)** Let $\vec{F}(x, y) = (0, -x)$ be a vector field on $\mathbb{R}^2$.

(a) Sketch the vector field $\vec{F}$.

(b) On your sketch in part (a), draw a curve C such that $\int_C \vec{F} \cdot d\vec{x}$ is negative. Explain briefly why the line integral is negative for the curve you have drawn.

(c) Calculate div $\vec{F}$.

(d) Calculate curl $\vec{F}$.

7. **(10 points)** The surface of a mountain is modeled by $h(x,y) = 25 - 2x^2 - 4y^2$. All distances are in miles. A hiker is walking on a path on this mountain. It begins to rain when she is at the point with $x = 1$, $y = 1$.

(a) In what direction should she head to descend the mountainside most rapidly? (In other words she would like to take the path which has the steepest descent.)

(b) Find the equation of the line tangent to the level curve through the point $(1, 1)$.

(c) Instead of descending most rapidly, the hiker decides to head off in the direction $\vec{i} + \vec{j}$. Find the rate of change in elevation in this direction.

8. **(9 points)** Evaluate the line integral $\int_C \vec{F} \cdot d\vec{x}$, where $\vec{F}(x, y) = (-y, x)$ and C is the closed path that consists of the line segment from $(-2, 0)$ to $(2, 0)$ followed by the semicircle of radius 2 centered at the origin in the upper half plane.