

Name: _____

Math 206: Fall 2011
Exam 2: 11/11/11

Write all your answers in your exam book. Label problems clearly and circle final answers. You may do the problems in any order. You do not need to simplify your answers, except where specified. Put your name on your exam and turn it in with your exam book.

For full credit you must show your work. Use correct vector notation even in your work. Good Luck!

1. (20 points) Let

$$\begin{aligned}r(u, v, w) &= uvw - u^2 - v^2 - w^2 \\u(x, y, z) &= y + z \\v(x, y, z) &= x + z \\w(x, y, z) &= x + y\end{aligned}$$

- (a) Find $\frac{\partial^2 r}{\partial u \partial v}$ and r_{vw} .
(b) Use the chain rule to find $\partial r / \partial x$ and $\partial r / \partial y$ in terms of x, y, z .
2. (20 points) Let \vec{F} be the vector field $\vec{F}(x, y, z) = (xe^y, z \sin y, xy \ln z)$. Find both the divergence and curl of this vector field.
3. (20 points) Let $\vec{f}: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be defined by $\vec{f}(x, y) = \left(\frac{2x}{y}, 3x^2 - xy + 5\right)$.

Suppose that $\vec{g}: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a differentiable function such that

$$\vec{g}(1, -1) = (3, 2) \quad \text{and} \quad D\vec{g}(1, -1) = \begin{bmatrix} 1 & -1 \\ 4 & 0 \end{bmatrix}.$$

Let $\vec{h} = \vec{f} \circ \vec{g}$.

- (a) Use the chain rule to find $D\vec{h}(1, -1)$.
(b) Use $\vec{h}(1, -1)$ and $D\vec{h}(1, -1)$ to find an approximation for $\vec{h}(0.99, -1.01)$.
4. (10 points) Write an equation of the plane tangent to the level surface, the ellipsoid $2x^2 + 4y^2 + z^2 = 45$, at the point $(2, -3, -1)$. **Simplify** your answer to $Ax + By + Cz = D$ form.
5. (10 points) Prove that the following function is continuous at $(0, 0)$.

$$f(x, y) = \begin{cases} \frac{1 - \cos(x^4 + y^2)}{x^4 + y^2}, & \text{if } (x, y) \neq (0, 0); \\ 0, & \text{if } (x, y) = (0, 0). \end{cases}$$

6. (20 points) Assume that the temperature in a room is given by the function

$$T(x, y, z) = e^{-x^2 - y^2} + e^{-x^2 + 4x - y^2 + 4y - 8} + z^2.$$

- (a) In what direction should a fly hovering at the point $(3, -1, 0)$ head so that the temperature is increasing most rapidly? **Simplify** your answer.
(b) In what direction should a fly hovering at $(3, -1, 0)$ head so that there is no change in temperature? (There are infinitely many answers, but you only need to give one answer.)