Math 105: Review for Exam II

1. Find dy/dx for each of the following.

   (a) $y = x^2 + 2^x + e^2 + e^{2x} + \ln 2 + \ln(2x) + \arctan 2$

   (b) $y = \sqrt{x} \cdot \arctan(5x)$

   (c) $y = \ln(\tan(2^{\cos(x^2)}))$

   (d) $y = \dfrac{x + e^{\pi}}{\cos 4 + \sin^5(6x)}$

2. Consider the curve defined by $x^3 + y^3 = \frac{9}{2}xy$ (known as the Folium of Descartes).

   (a) Find dy/dx.

   (b) Verify that the point (1,2) is on the curve above.

   (c) Find the equation of the tangent line at the point (1,2).

3. Evaluate the following limits.

   (a) $\lim\limits_{x \to 1} \dfrac{x^3 - 1}{7 - 7x}$

   (b) $\lim\limits_{x \to 0} \dfrac{1 - \cos 2x}{3^x}$

   (c) $\lim\limits_{x \to 0^+} x^2 \ln x$

   (d) $\lim\limits_{x \to 0} \dfrac{1 - \cos 4x}{5x^2}$

   (e) $\lim\limits_{x \to \infty} \dfrac{x^2}{2^x}$

4. Suppose that $y = f(t)$ is a solution to the differential equation $y' = \dfrac{1}{\pi} \arcsin t + y^2$ and that $f\left(\dfrac{\sqrt{2}}{2}\right) = \dfrac{1}{2}$. Find the equation of the tangent line to f at $\left(\dfrac{\sqrt{2}}{2}, \dfrac{1}{2}\right)$.

5. Find the following.

   (a) an antiderivative of $y = \dfrac{5}{\sqrt{1 - 9x^2}} + x^3 + \cos(2x) + e^3$

   (b) $\tan(\arccos x)$ (rewritten as an algebraic expression - no trigonometric functions)

6. Consider the function $f(x) = x^4 e^x$ with domain all real numbers.

   (a) Find the x-value(s) of all roots (x-intercepts) of f.

   (b) Find the x- and y-value(s) of all critical points and identify each as a local max, local min, or neither.

   (c) Find the x- and y-value(s) of all global extrema and identify each as a global max or global min.

   (d) Find the x-value(s) of all inflection points.

   (e) Sketch f.

7. How would your answers to the previous question change if the domain of f were $[-10, 10]$?

8. Use Newton's Method with an initial guess of $x_0 = -1$ to find the next three approximations to a solution of $x^3 + x + 1 = 0$. Then test your final approximation to see if it appears to be close to a root.

9. The rate of change of a population $P(t)$ of eels is proportional to the size of the population. When the population is 40000, it is growing at a rate of 400 eels per year. At time $t = 0$, the population is 10000.

   (a) Write a differential equation whose solution is $P(t)$.

   (b) Solve your differential equation.

   (c) When will the population reach 60000?

10. You are planning to build a box-shaped aquarium with no top and with two square ends. Your budget is \$288. If the glass for the sides costs \$12 per square foot and the opaque material for the bottom costs \$3 per square foot, what dimensions will maximize the volume? Be sure to show how you know you have found the maximum.