Math 205B Test 2 (45 points)

Name: ______________________________

- Check that you have 6 questions on three pages.
- Show all your work to receive full credit for a problem.

1. (8 points) Let A and B be 3×3 matrices, with $\det A = 2$ and $\det B = -5$. Use properties of determinants to compute:

   (a) $\det 4B$

   (b) $\det C$ where C is obtained from A by interchanging rows 1 and 3

   (c) $\det A^T B^T$

   (d) $\det (AB)^{-1}$ if AB is invertible. Otherwise, explain why AB is not invertible.

2. (6 points) Determine if each of the following sets is a subspace of the appropriate vector space. **If so, find a basis and dimension of the subspace.**

   (a) Let $W = \left\{ \begin{bmatrix} a - b + 2c \\ 2b + 2c + d \\ 4b + 4c + 2d \\ a + b + 4c + d \end{bmatrix} : a, b, c, d \text{ are real numbers.} \right\}$. Is W a subspace of $\mathbb{R}^4$? Explain.

   (b) Let $W = \left\{ \begin{bmatrix} a + b \\ b - 2 \\ b + 1 \end{bmatrix} : a, b \text{ are real numbers.} \right\}$. Is W a subspace of $\mathbb{R}^3$? Explain.

3. (8 points) Let $A = \begin{bmatrix} 32 & 6 & -10 \\ 1 & 33 & -5 \\ 4 & 12 & 10 \end{bmatrix}$ and $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation given by $T(\vec{x}) = A\vec{x}$. If possible, find a basis $\mathcal{B}$ for $\mathbb{R}^3$ consisting of the eigenvectors of A and write the $\mathcal{B}$-matrix for T. If such a basis does not exist, explain why. (The eigenvalues of A are 15 and 30.)

4. (a) (5 points) Find the characteristic polynomial of the matrix $\begin{bmatrix} 3 & 2 \\ 4 & 5 \end{bmatrix}$ and then find all the eigenvalues by solving the charateristic equation.

(b) (3 points) The vector $\begin{bmatrix} -7 \\ 7 \end{bmatrix}$ is an eigenvector of the matrix $\begin{bmatrix} 2 & 4 \\ 1 & -1 \end{bmatrix}$. Find the corresponding eigenvalue.

(c) (4 points) Find a non-zero vector in Nul A where $A = \begin{bmatrix} 1 & 0 & -3 & 2 \\ 0 & 1 & -5 & 4 \\ 3 & -2 & 1 & -2 \end{bmatrix}$.

(d) (3 points) Let $\vec{b}_1 = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$, $\vec{b}_2 = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$, and $\vec{b}_3 = \begin{bmatrix} 0 \\ 2 \\ -3 \end{bmatrix}$ be vectors in $\mathbb{R}^3$. Then $\mathcal{B} = \{\vec{b}_1, \vec{b}_2, \vec{b}_3\}$ is a basis for $\mathbb{R}^3$. If $[\vec{x}]_{\mathcal{B}} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$, then find $\vec{x}$.

5. (4 points) Suppose A is a 8×11 matrix. What is the largest possible rank of A? What is the smallest possible dimension of Nul A? Explain.

6. (4 points) Let $\mathcal{B} = \{\vec{v}_1, \vec{v}_2\}$ be a basis for a subspace H of $\mathbb{R}^5$. Let $\vec{x}$ be a vector in H that is not in the basis. Is the set $\{\vec{v}_1, \vec{v}_2, \vec{x}\}$ linearly independent? Explain.